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To: EDGES Group

From: Alan E.E. Rogers
Subject: Effects of digitizer non linearity

In memo #36 it was shown that digitizer non-linearity produces errors which are centered on the sources of RFI. For example, a spectrometer using 3-position switching of a 5 MHz bandpass stepped from 100 to 200 MHz will have systematics introduced whenever there is RFI within a 5MHz section. This means that even at the quietest sites a 5 MHz band centered on the 137-138 MHz Orbcomm band will be corrupted. In a wideband sampled system the non linearity corrupts the entire spectrum but it does so in a manner that may be acceptable. First the non-linearity produces harmonics and aliased harmonics of the RFI as well as intermodulation products between different sources of narrowband RFI. Second the non-linearity corrupts the calibrated spectrum in a manner that varies with the smoothness of the bandpass. From a simplified analysis:

$$T_{cal}\left[\frac{p_0 - p_1}{p_2 - p_1}\right] + T_L = T_A - \alpha T_A + \beta T_A / g$$

where p_0, p_1 and p_2 are the power spectra on antenna, load and load plus calibrator respectively. α *and* β are small positive constants which account for the non-linearity. For a 10% non-linearity at RFI power 26 dB over the noise total power

$\alpha \sim 0.2$
$\beta \sim 0.01$

α scales with the non-linearity and β scales as the non-linearity squared.

The linearity is defined as peak deviation from linear
e.g. a sine function $\sim x - x*x*x/6 = x(1 - x*x/6)$
when $x = 0.24 x*x/6 = 0.01$ 1% non-linear
α distorts the spectrum – but only be rescaling it
β =1e-4 for 1% non-linearity from the RFI and scales with the non-linearity squared.
The term in β is a more serious problem as it depends inversely on the bandpass g (with g=1 for a constant bandpass). If the bandpass is smooth and is well fit by the polynomial used to fit to T_a then the effect is no different than α.
If a ADC has THD – total harmonic distortion below -50 dB the non-linearity should be better than 0.5% and the systematics should be very low. For example if the bandpass has 1 dB ripple not fit by the polynomial and the non-linearity is 0.5 % then the β term gives

$$\beta * (1/g) * T_a = 2.5e - 5 * 0.25 * 1e3 \sim 6mK$$

For a bandpass ripple of 2 dB peak to peak and a Ta of 10^3K I obtained the following results from simulations using the gsl-rng random number generator to generate Gaussian signals.

Non-linearity	Rms deviation
14%	10K
8%	2K

Figure 1 shows the spectrum of Ta for 8% non-linearity on a log scale and figure 2 shows the spectrum after removing the simulated RFI and fitting a 7 term polynomial to a frequencies from 60 to 180 MHz. Figure 3 shows the spectrum for a 1% non-linearity after fitting a polynomial.

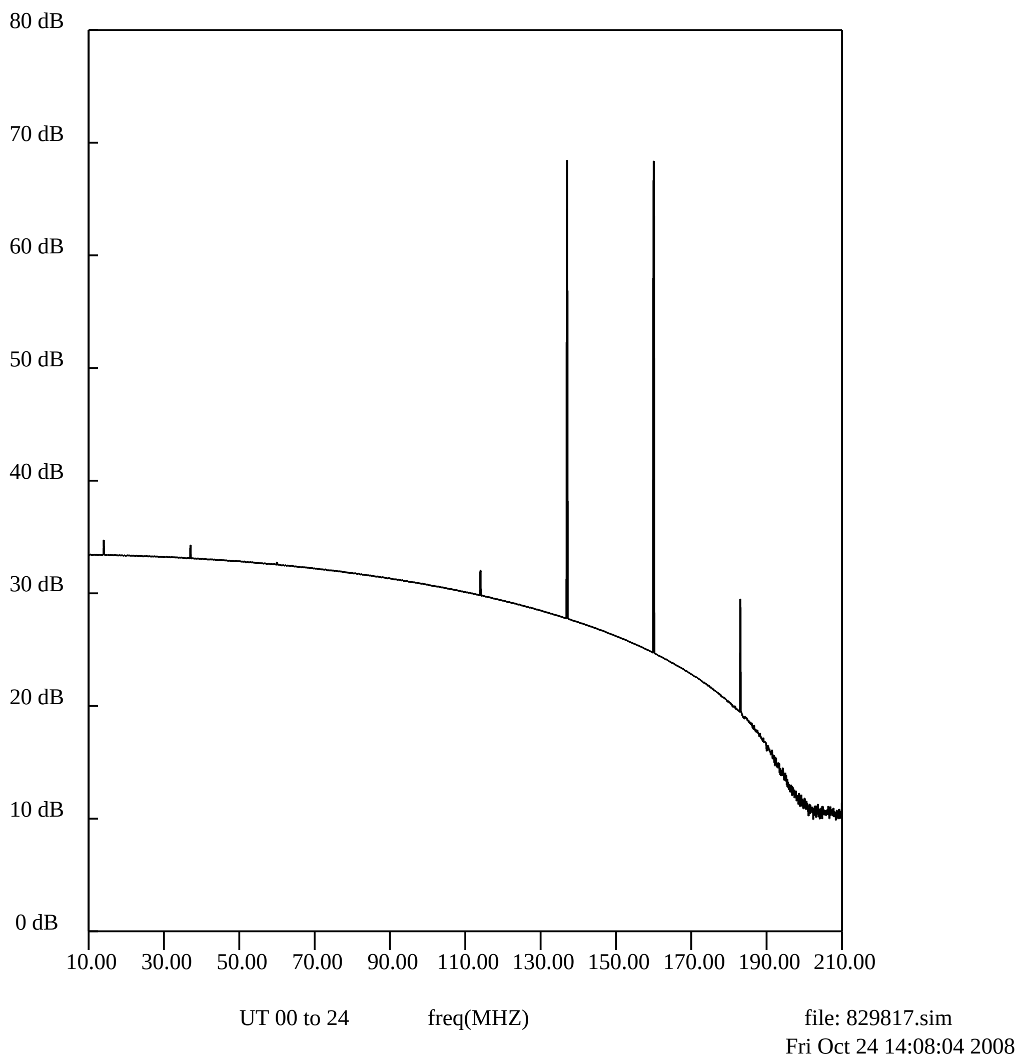

Figure 1.

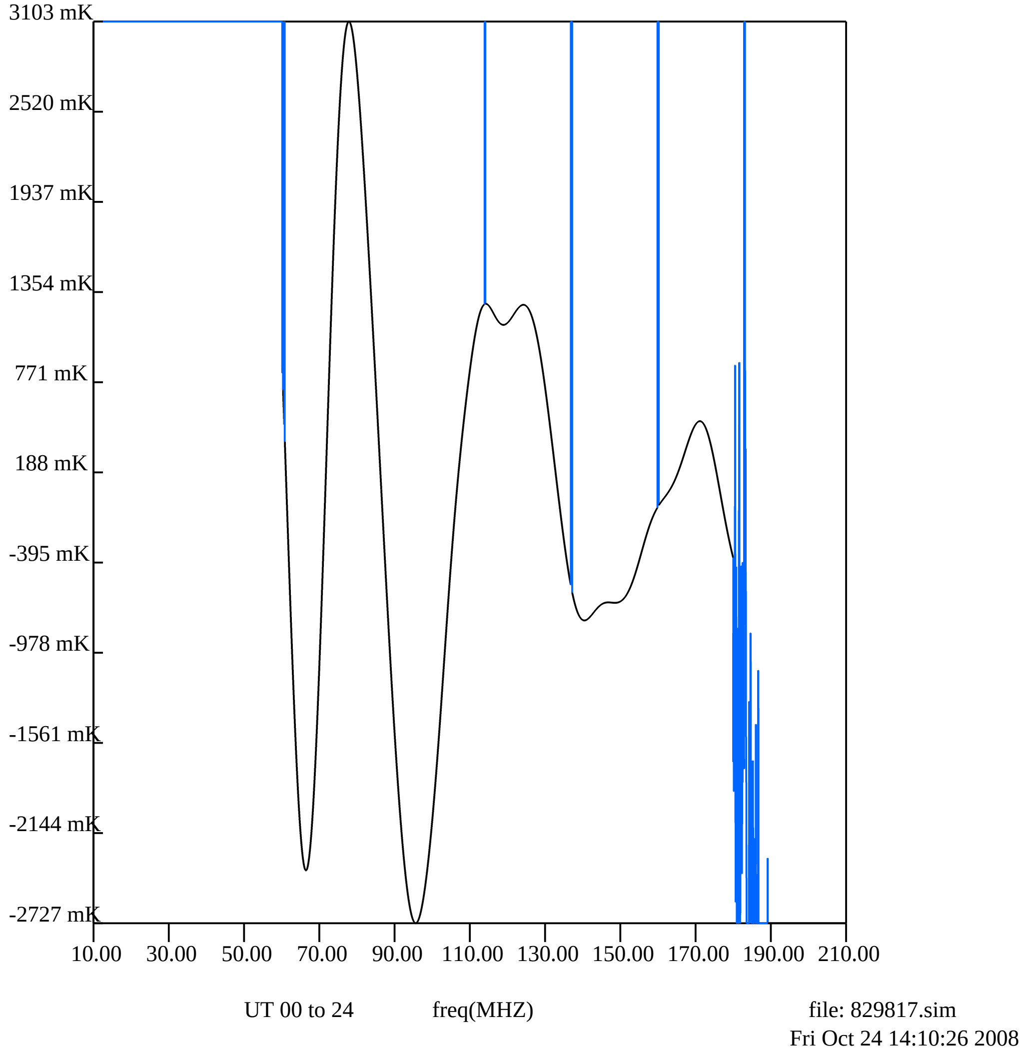
3103 mK
2520 mK
1937 mK
1354 mK
771 mK
188 mK
-395 mK
-978 mK
-1561 mK
-2144 mK
-2727 mK
10.00 30.00 50.00 70.00 90.00 110.00 130.00 150.00 170.00 190.00 210.00
UT 00 to 24
freq(MHZ)
file: 829817.sim
Fri Oct 24 14:10:26 2008

Figure 2.

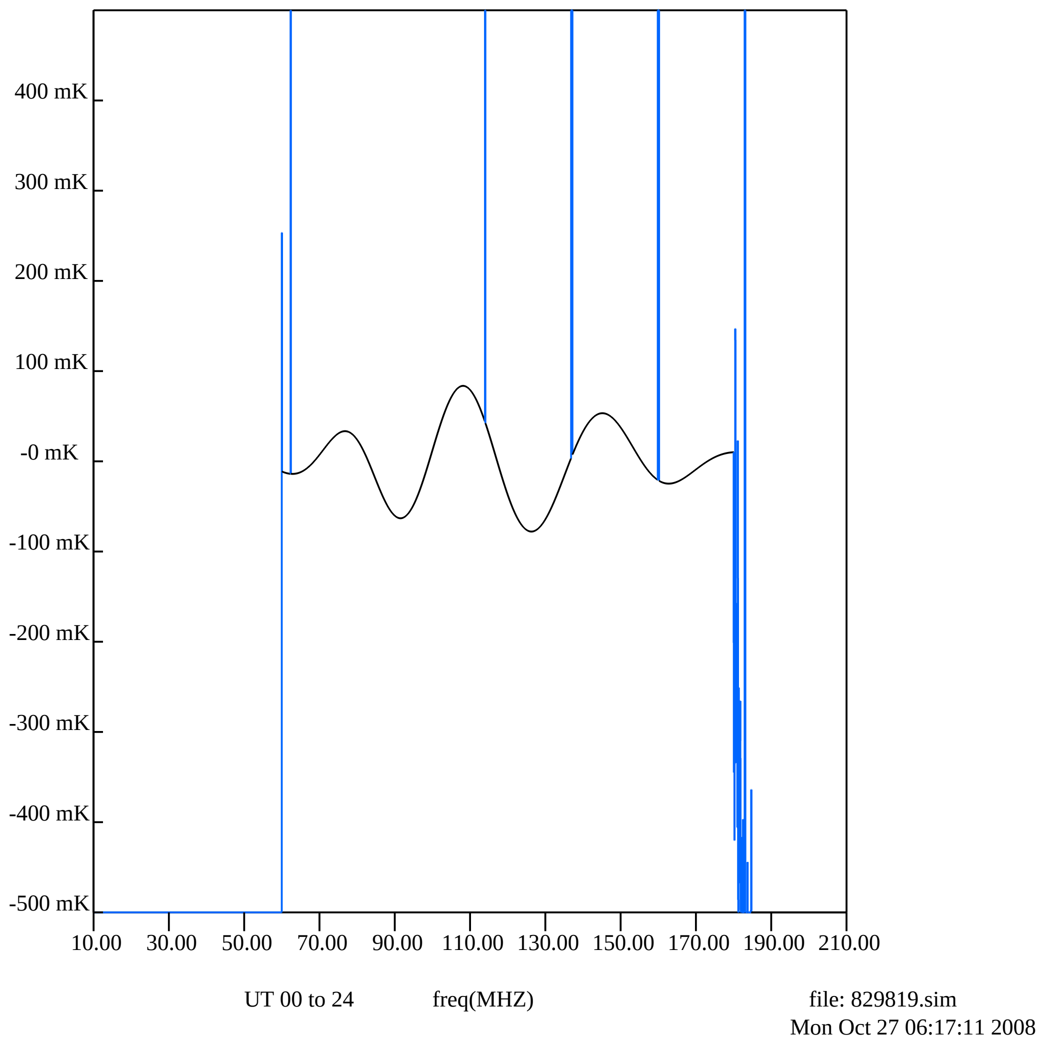
400 mK
300 mK
200 mK
100 mK
-0 mK
-100 mK
-200 mK
-300 mK
-400 mK
-500 mK
10.00 30.00 50.00 70.00 90.00 110.00 130.00 150.00 170.00 190.00 210.00
UT 00 to 24
freq(MHZ)
file: 829819.sim
Mon Oct 27 06:17:11 2008

Figure 3.