EDGES MEMO #064

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To: EDGES Group
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Subject: Model fitting functions

Currently EDGES analysis uses polynomial fitting. The problem with using polynomials is that they "soak-up" the EoR signature as the EoR becomes more gradual.

In memo #61 and 63 simulations show the limits on how far EDGES might go in detecting EoR signatures of increasing delta_z.

In order to improve EDGES ability to detect or exclude EoR signatures of sufficient delta_z to be of cosmological interest a new approach is considered. This memo gives the results of initial simulations of an approach in which polynomials are replaced with functions related to the hardware.

The new approach starts with measurements of the complex antenna impedance, a model of the LNA (see memo #62) the 408MHz full sky map and the NEC model of the antenna beam pattern. The following hardware based functions were chosen for a weighted least squares fit:

1] Sky noise model
The antenna temperature for a given integration is estimated using 408 MHz map scaled by the nominal spectral index of -2.5. The spectral index of the antenna temperature becomes changed by up to about 5% by the beam pattern when the Galactic center is above the horizon. The real part of the antenna impedance appears to the LNA to have a temperature equal to the antenna temperature in the combined model of the antenna and LNA.
2] Ground pick-up
The function representing the incremental change in the sky model when ground noise and thermal noise in the balun is added.
3] The incremental change with change in spectral index.
4] A change in FET transconductance
5] A change in FET gate-drain capacitance
6] and 7] a change in antenna impedance
8] a change in antenna delay
9] EoR signature

The antenna impedance as assumed to have been measured with a network analyzer in its location. Parameters 6,7 and 8 are needed to take-up an inaccuracy in these measurements. The LNA parameters are first "tuned" to match measurements made with a low loss open transmission line connected to the 3-position switch in place of the antenna. Parameters 4 and 5 are needed to account for any error in the LNA model.

The overall stability of the spectrometer is maintained by the 3-position switching in front of the LNA.

Simulation results

Simulations were run in which the model was perturbed by changing the antenna impedance, delay as well as the FET parameters and sky model hour angle. Table 1 gives the results for various scanned bandwidths and EoR delta_z.

		Accuracy required				
delta_z	bw (MHz)	ant.(ohm)	ant.delay (ns)	FET %	Spec Index%	SNR
0.1	20	50	10	30	50	15
1.0	20	50	2	30	50	6
2.0	20	5	0.3	10	50	1
0.1	50	5	0.2	10	50	20
1.0	50	3	0.2	5	50	15
2.0	50	3	0.2	5	30	10
3.0	50	3	0.1	5	20	6

Table 1. Accuracy requirements. The SNR is the value obtained for 1mK noise in 1MHz.

The simulations show little if any sensitivity to the following:

1] Difference in impedance between calibration on and off.
2] Error in the assumed value of noise cal. This error mainly scales the spectrum.
3] Frequency slope in noise cal.

Algorithms used

1] Weighted least squares

a. Parameter solution

$$\hat{x} = \left(A^T w A\right)^{-1} w y$$

Where $\hat{x}$ = best fit parameter

y = data matrix

A = design matrix

w = weight matrix

b. SNR estimate

$$SNR = \left(\hat{x}_{eor} \Big/ \left(rms \times \mathrm{cov}_{eor}^{1/2}\right)\right)$$

Where $\hat{x}_{eor}$ = the amplitude of the EoR signature

rms = the rms of residuals to fit

cov_{eor} = the covariance element from the inverse matrix

2] Noise

All the sensitive component of circuit elements are assumed to generate uncorrelated thermal noise at ambient temperature That is there is a voltage source of

$$e = \left(2KTBR\right)^{1/2}$$

In series with each resistive element of R ohms.

The real part of the antenna impedance acts as a noise source equal to the “antenna temperature.”

3] 3-position switch

3-position switching between the antenna, a 50 ohm load and 50ohm load with added noise is used to take out the added noise and bandpass of the spectrometer.

4] Transformation between impedance and reflection coefficient

The standard transformation

$$\Gamma = \left(z - z_0\right) / \left(z + z_0\right)$$

Where z = impedance

z_0 = 50ohms

Γ = complex reflection coefficient

In summary the new approach is to model all the “systematic” with physical model functions rather than polynomials. A byproduct of the approach will be more accurate measurements of the sky noise spectral index and a better understanding of the instrument.