EDGES MEMO #086

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
HAYSTACK OBSERVATORY
WESTFORD, MASSACHUSETTS 01886

March 29, 2012

Telephone: 781-981-5407
Fax: 781-981-0590

To: EDGES Group

From: Alan E.E. Rogers

Subject: Methods for improved calibration of VNA.

It may be possible to improve the accuracy of a VNA measurement of S11 in the 50 to 200 MHz range by improving the calibration via measurements of an open, shorted and terminated cable. The motivation for this work is the observation that while VNA measurements using the Agilent N5222A with 1-port electronic calibration are highly repeatable at the level of 0.001 linear uncertainty the specified accuracy of 0.01 is not sufficient for an EDGES detection of the hydrogen 21-cm line at 75 MHz.

1] Accuracy needed

If we assume the accuracy of EDGES is limited by the fractional accuracy of $\left(1-\left|\Gamma_{ant}\right|^2\right)$, where Γ_{ant} is the antenna reflection coefficient, then the fractional accuracy of $\left|\Gamma_a\right|$ of 0.0025 is needed to detect a 100 mK signal from hydrogen assuming a -20 dB return loss $\left(\left|\Gamma_a\right| \approx 0.1\right)$ for the antenna and 2000 K foreground.

2] Method for acquiring VNA corrections

The proposed method is to measure a long cable whose reflection coefficients can accurately calculated to derive instrumental directivity (e00), mismatch (e11), and tracking (e10) parameters which related the measured reflection, ρ, to the true reflection coefficient, σ via

$$\rho = e00 + e10\,\sigma/\left(1 - e11\,\sigma\right)$$

From a least squares fit to the VNA measurements. It may also be useful to include a measurement of a very accurately measured load since at the low frequencies of interest a load with good match to 18 GHz can be more accurately characterized by a measurement of its D.C. resistance.

3] Coax line reflection coefficient

The coax reflection coefficient can be accurately modeled from the solutions to telegrapher's equations:

$$\gamma = \left[\left(jwL + R\right)\left(jwC + G\right)\right]^{1/2}$$

$$z_c = \left[\left(jwL + R\right)/\left(jwC + G\right)\right]^{1/2}$$

L, C, R and G can be calculated from the dimensions of the inner and outer conductors, the relative permittivity and loss tangent of the dielectric, the conductivity of the conductors and the skin depths. While the capacitance, C per unit length is almost independent of frequency the inductance increases at low frequencies where the magnetic field within the skin depth of the conductors is significant. If *a* and *b* and the radii of the inner and out conductors

$$L_o = (\mu_o/2\pi)\ln(b/a)$$
$$C = 2\pi\varepsilon/\ln(a/b)$$
$$\delta = (\pi\,\sigma\,f)^{-1/2}$$
$$L_a = \mu_o\delta/(4\pi a)$$
$$L_b = \mu_o\delta/(4\pi b)$$
$$L = L_o + L_a + L_b$$
$$R = 2\pi f\,(L_a + L_b)$$
$$G = 2\pi fd$$
$$\mu_o = 4\pi \times 10^{-7}$$
$$\varepsilon = 8.85 \times 10^{-12}\ \varepsilon r$$
$$\sigma = 5.96 \times 10^{7} \qquad \text{for copper}$$

Loss tangent $d = 2 \times 10^{-4}$ for teflon

The values of are given in the limit for which a/δ and b/δ are much greater than 10. For 0.141 semi-rigid this approximation results in an error in the dispersion under 1% at 50 MHz. The exact expressions involve complex Bessel functions and can be found in Ramo and Whinnery, Fields and Waves in Modern Radio (Wiley 1953). The dielectric constant of Teflon which is nominally 2.1 may decrease with a slope of 0.05%/100 MHz at 100 MHz.

Tests indicate that 0.141 semi-rigid cable is probably better than even the most expensive instrument test flexible cables since uniformity of prime importance. A systematic, but constant error in impedance can also be obtained from the measurements as long as the instrumental errors e00, e10 and e11 are constant with frequency over the range of interest.