EDGES MEMO #091

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Subject: Simulation of EDGES.2 systematic error

The following measurements are needed for accurately calibrated data from EDGES:

1] VNA Data

S11a	S11 of antenna
S11ℓ	S11 of LNA
S11c	S11 of cable used to measure LNA noise waves
S11h	S11 of hot load
S11m	S11 of ambient load

2] Spectra from 3 position switch

sa	spectrum from antenna
sc	spectrum from cable to measure LNA
sh	spectrum from hot load
sm	spectrum from ambient load

To reduce the noise on the S11 data and provide values for all frequencies in the spectra the S11 data are fit with a Fourier series using least squares.

The first step is to use the hot and ambient load spectra to correct errors in the spectra due to error in internal calibration noise diode and error of internal load match.

The hot load and ambient load spectra are "inverted" using a function $f_{inv}(\ )$ which is the inverse of equation (13) of Rogers and Bowman

$$t_{hot} = f_{inv}\left(S11h, ss\ell, sh, \ell na3\right)$$
$$t_{amb} = f_{inv}\left(S11m, ss\ell, sm, \ell na3\right)$$

Where $\ell na3$ is an array of the LNA noise wave amplitudes. Using the assumption that each point in the inverted spectra is an error by a scale factor, sca, and an offset, ofs, these are derived as follows:

$$sca = \left(t_{hot} - t_{amb}\right)/\left(hot - amb\right) \text{ and } ofs = t_{amb} - sca \times amb$$

Where hot and amb are the measured temperatures of hot and ambient loads. The spectra are then corrected

$$sc = f\left(S11, S11\ell, f_{inv}\left(S11, S11\ell, (s-ofs)/a, \ell na3\right), \ell na3\right)$$

Where $f(\ )$ is the function of equation (13). In this correction the actual values of $\ell na3$ are not critical because the hot and ambient loads are close to a perfect match. If necessary the process can be iterated to refine the values. These corrected spectra are then used to derive accurate values of the LNA noise wave parameters by least squares fitting of $\ell na3$ to the spectrum of the open ended cable.

$$t_{sky} = f_{inv}\left(S11a, S11\ell, sc, \ell na3\right)$$

Using simulated data based on a sky temperature of 500 K at 150 MHz with spectral index = -2.5 and representative values of the S11 coefficient and LNA noise wave parameters the following errors sensitivity coefficients were obtained for the band from 50 to 100 MHz:

Parameter	S11 error	Coeff A mK	Coeff B mK	Coeff C mK
Antenna amplitude	0.01 dB	189	24	23
Antenna phase	0.5°	568	111	31
LNA amplitude	0.01 dB	25	13	7
LNA phase	0.5°	406	86	33
Cable amplitude	0.01 dB	20	10	3
Cable phase	0.5	18	12	2
Hot load amplitude	0.1 dB	21	10	2
Hot load phase	5°	718	109	14
Ambient load amplitude	0.1 dB	8	7	2
Ambient load phase	5°	535	91	12
	Temp error			
Hot temperature	1 K	36000	1841	0
Ambient temperature	1 K	35790	2289	0
Change in internal comparison load	1 K	36290	2317	0

Table 1. Sensitivity of EDGES to errors in S11 and temperatures.

Coefficient A is the rms differences between the input model and the derived output based on the 4 measured spectra and 5 VNA measurements.

Coefficient B and C are the rms difference after solving for a sale factor and a scale factor, plus an offset respectively.

When the simulation is run for the frequency range 100 to 200 MHz the sensitivity to the S11 measurements of the antenna, LNA and loads for Coeff. A are reduced by a factor of about 6 owing to the lower sky temperature while the sensitivity to the S11 measurements of the cable used to measure the noise waves is about the same. However the values for Coeff. C are only reduced by a factor of about 2.

Comments on the results in Table

Extreme accuracy in the S11 measurements of the antenna and LNA are needed. The accuracy for S11 measurements of the hot and ambient appears less challenging until it is recognized that the accuracy that can be achieved with a VNA measured in dB and degrees of phase degrades rapidly for the reflection coefficients below -20 dB. The accuracy needed for the measurement of the phase of S11 can be reduced by improving the LNA match or by including an attenuator between the LNA and the antenna. Since an error in scale or even an error in scale plus a constant is acceptable in a search for the signature of the early hydrogen absolute accuracy as indicted by coefficient A is not needed. This is fortunate since the S11 accuracy needed to obtain absolute accuracy at the level of 30 mK is well beyond the capabilities of a VNA.

It should be noted that an error in the measurement of temperature only produced an error in scale plus offset. Also a change in temperature of the comparison load in the EDGES following its calibration with hot and ambient loads result in a change in scale and offset. EDGES contains a probe which produces a signal in the spectrum between 1 and 2 MHz which provides a continuous record of temperature via a measurements of its frequency. With this data the effect of the change of temperature of the comparison load can be corrected. In addition, if the temperature coefficients of the LNA S11 and noise wave parameters are measured in the laboratory the effects of these changes can also be corrected using the temperature data. Alternately the temperature of the EDGES electronics might be controlled. Fortunately the change of LNA reflection coefficient is small as shown in the following table of measurements.

	25 °C		33 °C		Difference	
Frequency MHz	Amp dB	Phase deg	Amp dB	Phase deg	Δ amp	Δ phase
50	-11.920	79.41	-11.896	78.86	0.024	-0.55
100	-18.621	-45.84	-18.44	-45.99	0.180	-0.15
150	-14.823	-136.26	-14.728	-135.79	0.095	0.47
200	-11.309	174.68	-11.256	175.04	0.053	0.36

Table 2. Measured LNA reflection coefficient at 25 and 30 °C.

Conclusions

The simulations of systematic errors show that VNA accuracy is critical. An accuracy of 0.01 dB and 0.5 degrees for S11 measurements is needed to reduce the systematic signature below 50 millikelvin after allowing an error in scale factor and a constant offset. While some initial tests show this level of accuracy might be achieved with heavy, expensive high end VNAs it's not clear that this accuracy can be achieved in the field with a portable VNA. Given the stability, low temperature sensitivity and stability of the EDGES LNA and noise diode it may be possible to rely on the S11 measurements, except those of the antenna, taken under controlled conditions in the laboratory. But this still requires accurate VNA measurements of the antenna to be made in the field.

If sufficient accuracy cannot be realized with a portable VNA it may be necessary to transport a large VNA to the field. Costs might be reduced by using an old HP8515A available at a tenth of the cost. If the performance is limited by the size and quality of the directional coupler the old instruments may be as accurate as the new Agilent PNAs. Another possibility is to construct the antenna with such precision that it can be measured, taken apart and reassembled to have the same S11.

Reference

Rogers, A.E.E., Bowman, J.D., (2012) Absolute calibration of a wideband antenna and spectrometer for accurate sky noise temperature, *Radio Sci.***47**, RSOK06, doi: 10.1029/2011RS004962.