EDGES MEMO #112

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To: EDGES Group

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Subject: Using EDGES to correct changes in antenna reflection

When a long cable is added between the antenna and the LNA errors in the assumed or measured values of reflection coefficient result in structure in the calibrated spectrum with a periodicity of the inverse delay and its harmonics. While this has been already noted (see memo 111) it has yet to be fully explored. The theoretical basis comes from the expression for the spectrum prior to correction for the reflection coefficients. In simplified form

$$P = \left(T_{sky}\left(1-\left|\Gamma_a\right|^2\right)+T_u\left|\Gamma_a\right|^2\right)\left|F\right|^2 + T_c\left|\Gamma_a\right|\left|F\right|\cos\left(\phi+\phi_0\right)$$

Where T_{sky} = sky temperature from antenna

T_u = uncorrelated portion of noise from LNA and cable losses reflected back from the antenna.

T_c = correlated portion of reflected noise

Γ_a = antenna reflection coefficient

Γ_ℓ = LNA reflection coefficient

F = $\left(1-\left|\Gamma_\ell\right|^2\right)^{1/2} \Big/ \left(1-\Gamma_a\Gamma_\ell\right)$

ϕ = phase of $\left(\Gamma_a F e^{-iw\tau}\right)$

ϕ_0 = portion which varies slowly with frequency

τ = cable delay

w = 2π times frequency

Consider the case of $T_c = 0$. In this case the spectrum is modulated by the change of $\left|F\right|^2$ with frequency. This term is approximately $T_{sky}2\left|\Gamma_a\right|\left|\Gamma_\ell\right|\cos\left(w\tau+\phi_1\right)$

Where ϕ_1 is the phase of $\Gamma_a\Gamma_\ell e^{iw\tau}$ for small values of $\left|\Gamma_a\right|\left|\Gamma_\ell\right|$. If T_c is non zero there is an added contribution with period $1/\tau$ of $T_c\left|\Gamma_a\right|\cos\left(w\tau+\phi_2\right)$ and in addition a lower amplitude term with period $1/2\tau$.

$$\left(T_c/2\right)\left|\Gamma_a\right|^2\left|\Gamma_\ell\right|\cos\left(2w\tau+\phi_3\right)$$

In general the phases ϕ_0, ϕ_1 and ϕ_2 are different so the terms with period $1/\tau$ don't necessarily add in phase. In most cases the first of these terms dominates owing to the large sky contribution.

A simulation using the S11 for the Roberts balun antenna, LNA S11 of -20 dB, typical noise wave values and 80 ns cable delay results in a sinusoidal modulation with 12.5 MHz period of about 20 mK for a 0.01 dB or 0.1 degree error in antenna S11. A cable delay of 80 ns allows the inclusion of polynomials, for errors in the antenna S11 amplitude and phase, to the basis functions for the measurement of the EoR spectrum.

The following basic functions were used

Basis function #	Form	Purpose	Notes
0	f^{-s}	Scale	
1	1	Constant	
2	f^{-2}	Ionosphere emission	1
3	f^{-s-2}	Ionospheric absorption	1
4	$f^{-2.8+5\times10^{-3}\log(f)}$	Foreground synchrotron	2
5	$f^{-2.5+5\times10^{-5}\log(f)}$	Foreground free-free	2
6 to 6+2n	n^{th} term polynomial	S11 amp and phase	
6+2n	Gaussian	EoR model	

Table 1. Basis functions used in simulations. note 1 – see memo 79; note 2 – see memo 101, where s is the foreground spectral index (nominally 2.5).

Using these basis functions we obtain the covariance values in Table 2 for a frequency range from 90 to 195 MHz for a Gaussian EoR signature of 40 and 50 MHz full width at half power centered at 145 MHz. A value of the square root of covariance of 6 results in a detection with SNR of 10 for a 20 mK Gaussian and EDGES noise for 100 days of observing 8 hours/day.

	40 MHz	50 MHz	n
#terms	Covar ½	Covar ½	
1	1.0	1.0	
2	1.2	1.3	
3	1.6	1.9	
4	2.1	2.8	
5	4.5	7.6	
6	4.7	8.7	
7	5.5	8.7	0
9	5.6	9.3	1
11	5.6	9.8	2
13	5.8	10.8	3
15	5.9	11.1	4
17	6.3	11.8	5

Table 2. Increase in covariance with number of basis functions used.

The simulation results in Table 2 show that adding a polynomial with 4 terms for correction of S11 amplitude and phase results in only a small increase in covariance while most of the covariance increase comes from the introduction of basis functions needed to remove the effects of the foreground and ionosphere. Preliminary tests indicate that these functions will also remove the effects of the change in antenna beam with frequency without the need for additional functions. However modeling of the sky based on the Haslam map will be needed to make first order corrections.

While 80ns was used for the cable delay 40 ns is adequate for the frequency range 90-195 MHz. If it turns out higher order polynomials are needed to correct the antenna S11 then 80 ns may be required. 80 ns is needed for the frequency range of 50-100 MHz.