

**MASSACHUSETTS INSTITUTE OF TECHNOLOGY**  
**HAYSTACK OBSERVATORY**  
**WESTFORD, MASSACHUSETTS 01886**  
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*Telephone: 781-981-5400*  
*Fax: 781-981-0590*

To: EDGES Group

From: Alan E.E. Rogers

Subject: Noise analysis of an imperfect attenuator

The sky noise spectrum at 75 MHz is about 1500 K which is much greater than LNA noise so that there is an advantage to using an attenuator between the antenna and the LNA. This is discussed in memos 70 and 75 and in Rogers and Bowman 2012. The attenuator not only reduces the dynamic range required for the “low” band but also reduces the effects of errors all the S11 measurements except that of the magnitude of the antenna S11. However this advantage can only be realized for an attenuator that deviates from a perfect 50 ohm match on both ports by employing the following procedure:

- 1] Measure the antenna S11 without the attenuator.
- 2] Measure the attenuator loss separately using an average of open and shorted S11. Alternately for low frequencies the equivalent Tee or PI resistances from the resistance of each port with the other port open along with the resistance between ports.
- 3] For an accurate solution use a circuit model from the derived Tee resistances or estimate the loss factor L from the S11 open and short to provide a good approximation.

The attenuator can also be analyzed as a full 2-port network and measured with a 2-port VNA but the VNA accuracy is a severe limitation. For example an  $|S_{12}|$  error of only 0.01 dB corresponds to 3K out of 1500 K.

For an attenuator whose impedance is exactly 50 ohms the effective loss is

$$L_0 = L \left( 1 - |\Gamma|^2 / L^2 \right) / \left( 1 - |\Gamma|^2 \right)$$

$$\text{and } p = \left( L_0 T + (1 - L_0) T_{amb} \right) \left( 1 - |\Gamma|^2 \right)$$

where L = attenuator loss factor

$\Gamma$  = S11 of antenna measured through the attenuator

T = sky temperature for loss less antenna

$T_{amb}$  = temperature of the attenuator

p = noise power in kelvin

if the attenuators is not perfect a better approximation is to substitute

$$\Gamma = \Gamma_a L_{av}$$

Where  $\Gamma_a$  is the antenna S11 measured directly without the attenuator and  $L_{av}$  is the attenuator loss factor estimated from the average of half the open and shorted S11

$$L_{av} = \left( |\Gamma_{open}| + |\Gamma_{short}| \right) / 2$$

Figure 1 shows the deviation of the estimated power,  $p$ , from that computed using a circuit model for a nominal 6 dB attenuator with 50.2 ohm impedance. The antenna S11 is that of the EDGES fourpoint with an added delay of 200 ns to show how the impedance deviation introduces a dependence on the S11 phase. The rms deviation of the approximation based on separate S11 measurements of the antenna and the attenuator (or lossy cable) is 130 mK. If antenna S11 is measured through the attenuator and the standard loss calculation used the rms error increases to 800 mK for the 50.2 ohm impedance.

It is noted that almost all the deviation due to attenuator (or equivalently cable see memo #115) impedance error is the result of a dependence on the phase of S11. In this case averaging over frequency can be used, in the case of the broad spectral line, to reduce the effect by up to 2 orders of magnitude.

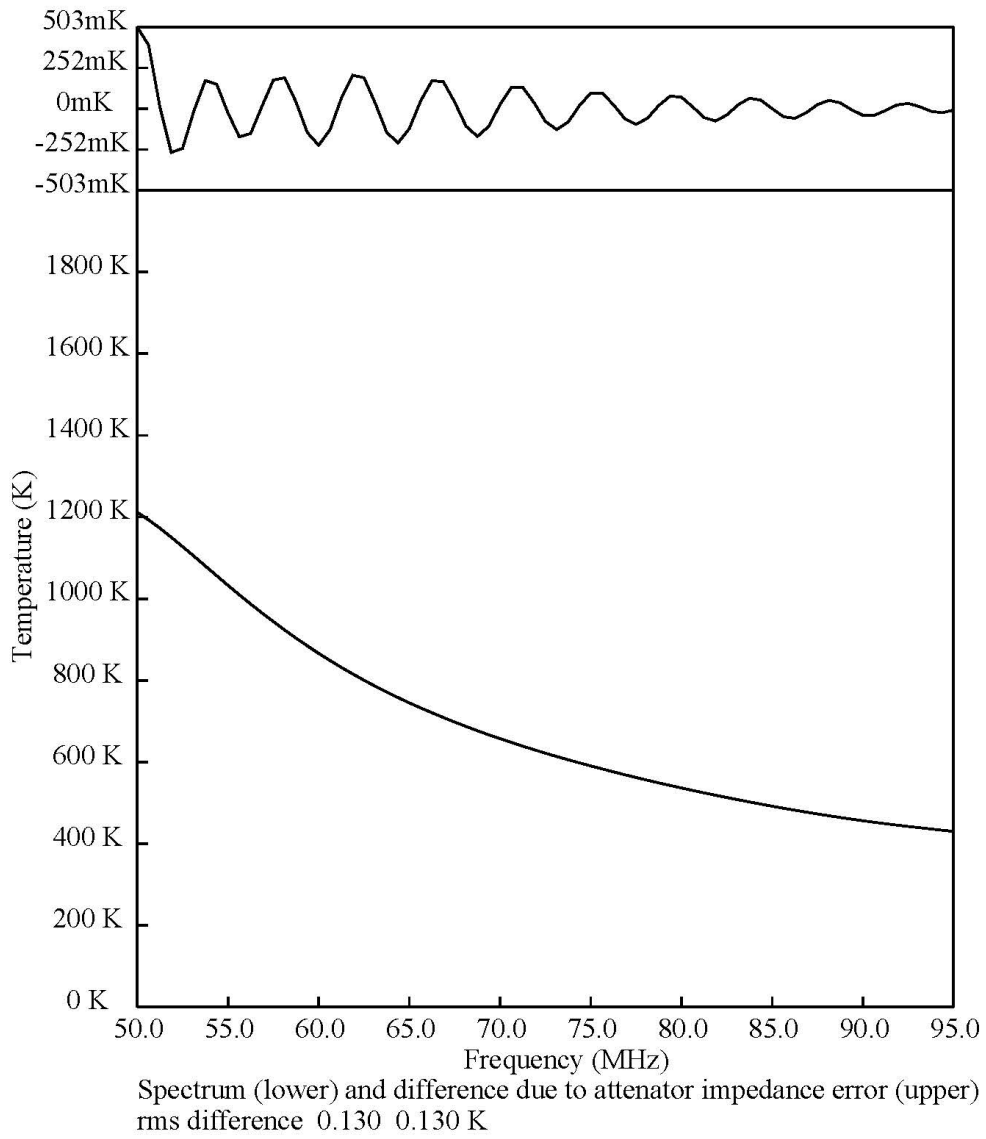


Figure 1. Effect of attenuator impedance error.