MASSACHUSETTS INSTITUTE OF TECHNOLOGY
HAYSTACK OBSERVATORY
WESTFORD, MASSACHUSETTS 01886

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Telephone: 781-981-5400
Fax: 781-981-0590

To: EDGES Group

From: Alan E.E. Rogers

Subject: Simulations of the removal of the foreground, ionosphere and frequency dependence of the beam.

In this memo the analysis of memo 101 is extended for a more complete simulation which includes the effects of refraction by the F-layer and the variation of antenna beam shape with frequency. These simulations use the following basis functions:

Basis number	Function	Purpose	Ref.
0	$f^{-2.5}$	Scale	
1	1	Constant	
2	$f^{-2.0}$	Ionos. Emission	Memo #79
3	$f^{-4.5}$	Ionos. Absorption	Memo #79
4	$\ell og(f) f^{-2.5}$	Foreground "gamma" change in spectral index.	
5	$f^{-2.8+5\times10^{-3}\log(f)}$	Point sources in foreground	Memo #101
6	$f^{-2.15+5\times10^{-5}\log(f)}$	Free-free foreground	Memo #101
7	$f^{-0.5}$	Antenna beam change with frequency	Memo #71
8	$e^{-0.69(f-f_0)^2/w^2}$	EoR signature	

To simulate the effects of the foreground and ionosphere the 408 MHz sky map was modified in spectral index, ionospheric emission, absorption and refraction was then convolved with the EDGES antenna beam. In more detail the sky noise was modified by ionosphere emission and absorption in the F-layer assuming an opacity at the zenith of 0.016 dB at 150 MHz with a frequency dependence of $\left(f - f_p\right)^{-2.0}$ and an elevation dependence of

$$\left(R+h\right)\left(R^2\sin^2 el+2Rh+h^2\right)^{-1/2}$$

Where R=6356 km, h=100 km and f_p=5 MHz. The emission assumes an election temperature of 1000K. Ionospheric refraction in the D-layer assuming 7×10^{-3} radians at the horizon of 100 MHz with a frequency dependence of $\left(f-f_p\right)^{-2.0}$ and an elevation dependence of

$\cos\left(el\right)\left(\sin^2 el+2h/R\right)^{-1.5}$ from Bailey, D.K. J. Terr. Magn Atmos. Elec. 53, 41, (1948).

In this case h=500 km and the horizon refraction corresponds to an electron density of 10^{12}m^{-3}.

In addition to the ionosphere the spectral index was taken -2.55 for Galactic latitudes within 20 deg of the equator and -2.45 elsewhere. The foreground was also modified by adding a frequency dependence to the spectral index of $-0.12\log\left(f/f_0\right)$. The frequency dependence of the antenna beam was taken from FEKO EM modeling of the EDGES Fourpoint dipole over a ground plane. The analytic expression for a dipole over a ground plane was also tested and found to make little difference to the results of the simulations.

Figure 1 (lower portion) shows the rms residuals to the data using only the first basis function to illustrate how the measured spectrum from a perfectly calibrated antenna and spectrometer deviates from a spectral index of -2.5. The curves labeled 0, 1, 2, 3 and 4 are for the cases of corruption by ionospheric opacity, spectral index variation, ionospheric refraction, frequency dependence of the beam and all effects combined respectively.

The site latitude is that of Boolardy -26.7 degrees and the frequency range considered is 55 to 95 MHz. The upper portion of the plot are the residuals to a fit using the 8 basis functions. In figure 2 the rms curves with the removal of only a scale change are repeated but now the upper plot shows the amplitude of the EoR signature, without any EoR signature added to the simulated spectra, obtained from a 9 parameter fit which includes the EoR signature assumed to be centered at 75 MHz with a full width at half power of 16 MHz. This shows that the set of basis functions are able to take out the effects of beam change, ionosphere and foreground except in the range of GST from about 14 to 22 hours. A similar plot for a latitude of 42° is shown in Figure 3. At both latitudes the effects on the EoR are a little lower for the dipole orientation which minimizes the horizon response in the NS direction.

The values assumed for the ionosphere were chosen to be representative of observations made between midnight and an hour before sunrise. Figure 4 (lower curve) shows the EoR signature after the removal of the ionosphere and foreground for a simulation of data with 100 mK absorption of 16 MHz full width at half power added. In addition noise is added to simulate 60 days of observations taken for 5 hours each day after midnight when the moon is below the horizon. The simulations do not include man made interference and assume perfect antenna and spectrometer calibration. While the basis functions largely remove foreground and ionosphere they also remove much of the EoR signature so that the standard deviation of the EoR is increased by a factor of 4.3 for a SNR of about 10 in the simulated detection of the EoR signature in 60 days. Table 1 shows the relative standard deviation changes with EoR signature width and frequency span.

EoR signature width MHz	Standard deviation	Frequency range MHz
12	2.6	55-95
16	4.3	55-95
20	7.2	55-95
16	2.6	50-100

Table 1: Standard deviations from the parameter covariance matrix.

The SNR in the absence of systematics is about 10 for EDGES integration for 300 hours for the standard deviation of the EoR estimate equal to 4.

Summary and conclusions

Figure 1,2 and 3 show that the effects of spectral index variation in the foreground along with change of beam shape with frequency are dominant and provided observations are made at night the measurements of the EoR signature should not be limited by the ionosphere. The additional monotonically decreasing functions of frequency introduced to take out the ionosphere result in only a small increase variance of the EoR in the variance-covariance matrix. The effects of ionospheric refraction are very small for the EDGES antenna which has no vertical currents forcing the response to decrease rapidly below a maximum of -20 dBi at 5 degrees elevation.

While the effects of the ionosphere are much smaller in the 110-190 MHz range the EoR 30 mK signature can only be detected in 30 days at an SNR of about 10 if the width is less than 40 MHz. again assuming perfect noise limited instrumentation.

Further, the basis functions needed to take out foreground and ionosphere, also take out some broadband monotonic errors in instrument calibration and reflection coefficients.

Added note on the effect of a horizon mask:

The simulation was also run with a horizon mask with an assumed thermal temperature of 300 K. For the EDGES antenna a horizon up to 10 degrees elevation makes little difference to the ability to extract the EoR signature as the change in spectrum is smooth and is absorbed by the same basis functions. The total power contribution from the ground is only 0.4% or 1.2 K for a 10 degree horizon. A horizon of 10 degree is expected to reduce the RFI by a substantial factor. A similar test in the 110 to 190 MHz range shows that a horizon up to 7 degrees elevation has little effect on the EoR extraction of a 30 mK signature with 40 MHz width. A 10 degree horizon might be tolerated if it only goes to 10 degrees in a direction aligned with the dipoles minimum horizon response.

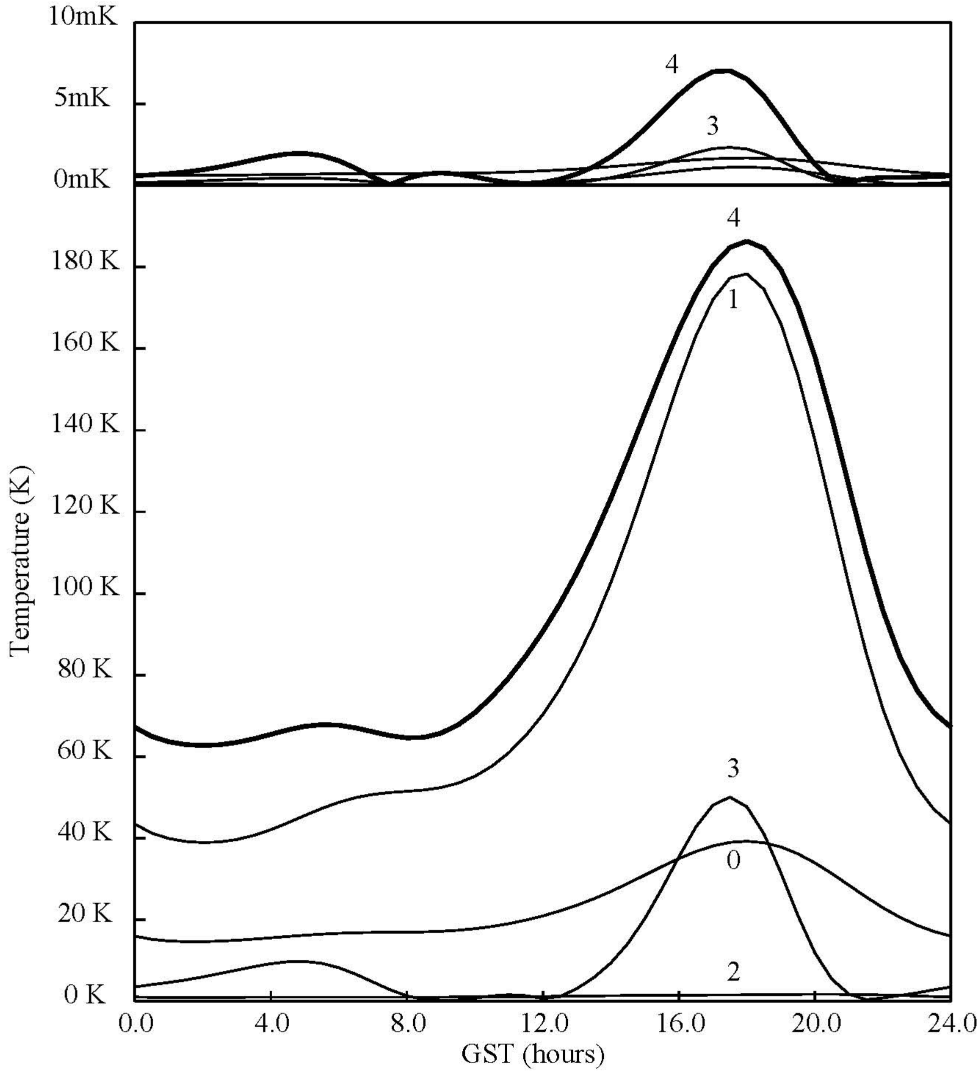

Figure 1. Lower curves are the rms deviations from a nominal spectral index of -2.5 for the ionosphere, spectral index, ionospheric refraction and frequency dependence of the beam and all combined labeled 0, 1, 2, 3, 4 respectively. The upper plot are the rms deviations after removing the 8 basis function.

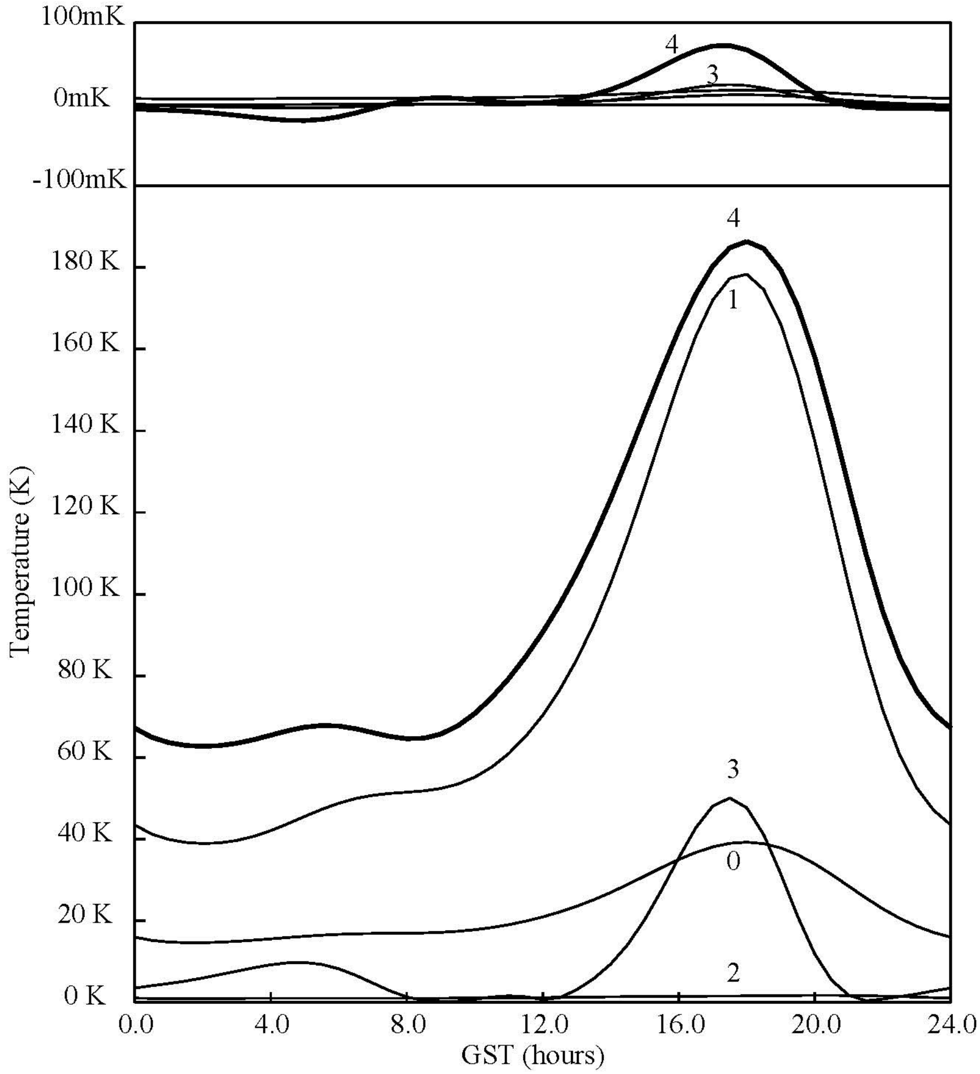

Figure 2. Lower curves are same as in figure 1. Upper curves are the values of EoR signature bias introduced by the lack of ability of the basis functions to accommodate the deviations of the foreground from an ideal uniform spectral index of -2.5. The curves labeled 3 and 4 are for the effect of the antenna beam variation and all effects combined respectively.

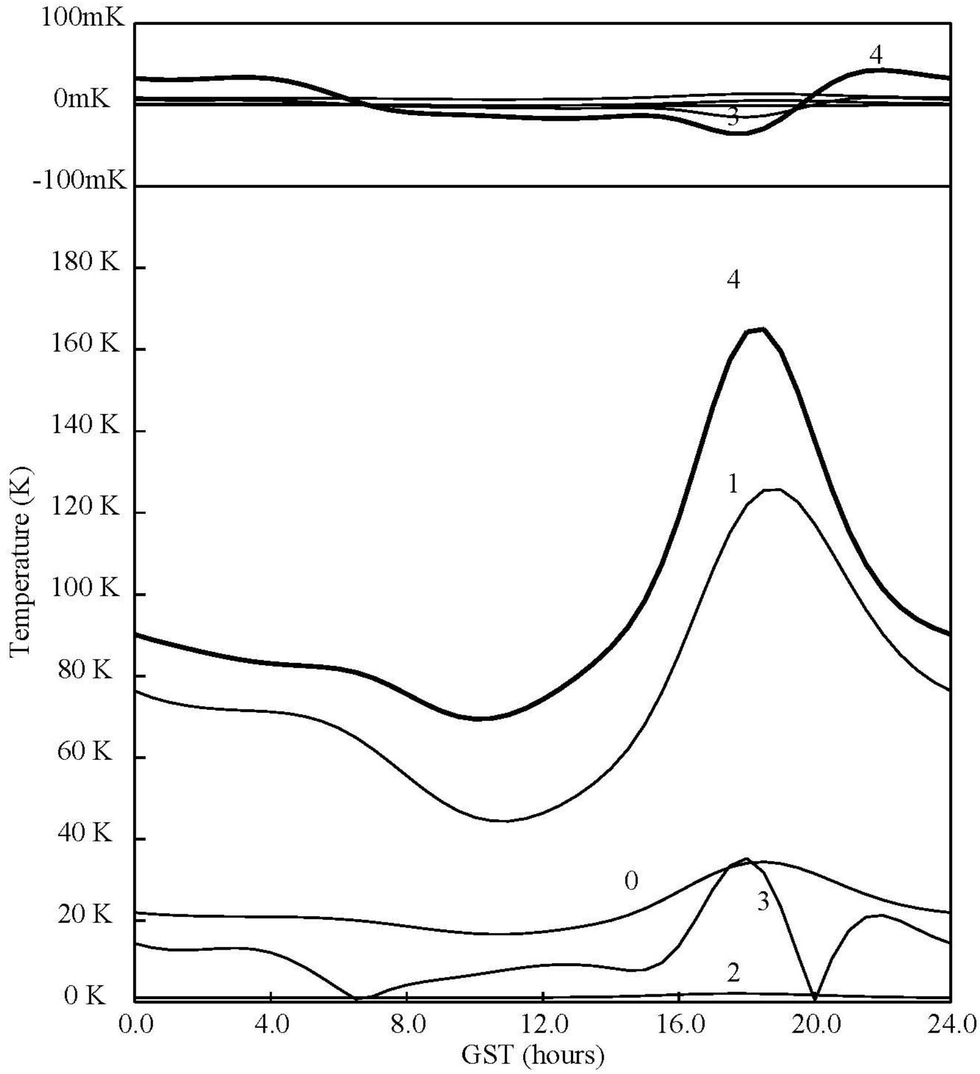

Figure 3. Same as figure 2 for a latitude of 42 degrees. For both latitudes the performance is slightly better with the dipole oriented NS.

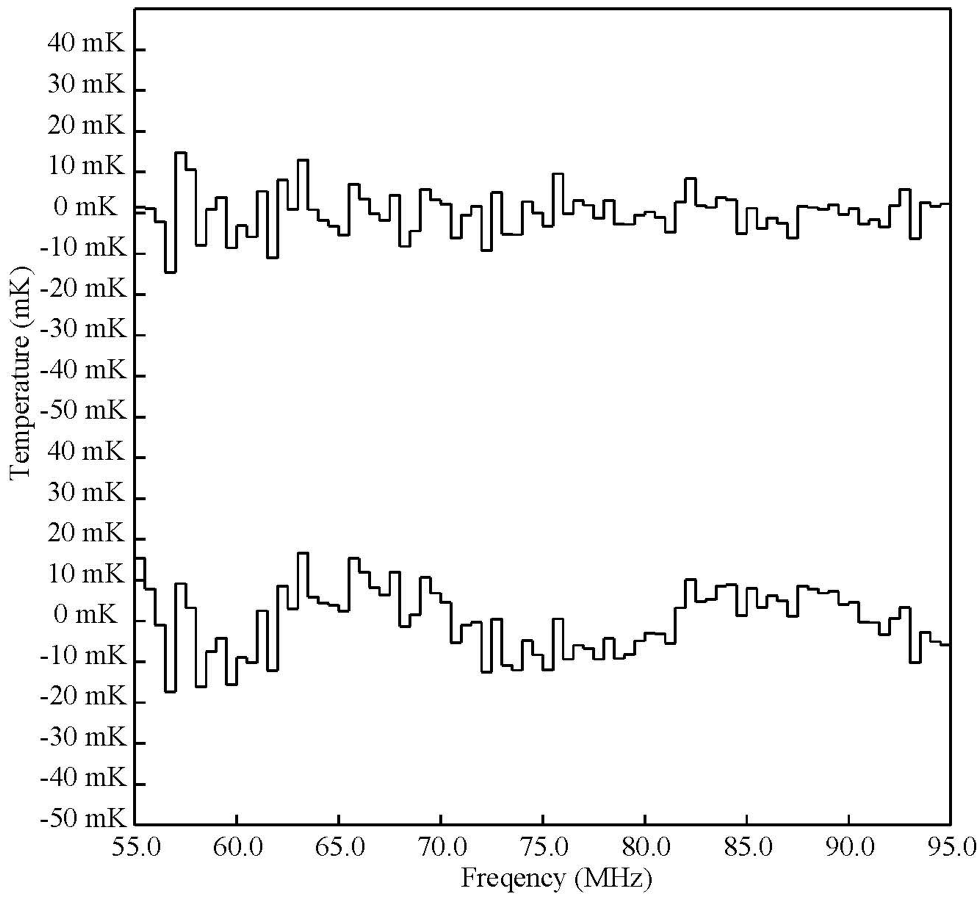

Figure 4. The lower curve shows the residuals to the best fit using basis functions 0 through 7 while upper curve shows the residuals for the best fit which includes basis function 8. This shows that the 100 mK absorption is not completely removed by the basis functions needed to remove the foreground, ionosphere and frequency dependence of the beam.