EDGES MEMO #122

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Subject: Effect of VNA phase errors

In several memos the effects of VNA errors have been evaluated from simulations. Greater insight is obtained from the expression for the signal power:

$$\left[T_{sky}\left(1-\left|\Gamma_a\right|^2\right)L+T_u\left|\Gamma_a\right|^2L^2+T_a\left(1-L\right)\left(1+\left|\Gamma_a\right|^2L\right)\right]\left|F\right|^2+T_c\left|\Gamma_a\right|\left|F\right|L\cos\phi$$

Where T_{sky} = sky temperature (1700 K low band 300 K high band)

T_a = ambient temperature (290 K)

T_u = LNA uncorrelated wave (80 K)

T_c = LNA correlated wave (20 K)

Γ_a = antenna S11 (0.2)

Γ_e = LNA S11 (0.1)

L = attenuation loss factor

$F=\left(1-\left|\Gamma_e\right|^2\right)^{1/2}\Big/\left(1-\Gamma_a\Gamma_e L\right)$

ϕ = phase of F

Dividing by L we rescale to get

$$\left[T_{sky}\left(1-\left|\Gamma_a\right|^2\right)+T_u\left|\Gamma_a\right|^2L+T_a\left(1-L\right)\left(1/L+\left|\Gamma_a\right|^2\right)\right]\left|F\right|^2+T_c\left|\Gamma_a\right|\left|F\right|\cos\phi$$

The rescaled expression makes it clear which terms are effect by attenuation. Since T_c is much less than the antenna temperature the sensitivity to VNA phase errors is dominated by change in the magnitude of F. The fractional error in $\left|F\right|^2$ is approximately

$$2\left|\Gamma_a\right|\left|\Gamma_e\right|L\Delta\phi$$

For a phase error of 0.06° this is about 4×10^{-6} or 4×10^{-5} with and without 10 dB attenuation respectively using the typical values in parentheses. The use of 10 dB attenuation reduces the sensitivity to VNA phase error of 0.06 degrees to 7 mK for 1700 K antenna temperature. The contribution from the correlated noise wave is negligible. In the high band the use of attenuation increase the sensitivity to VNA S11 magnitude error because the ambient noise reflected from the antenna dominates. The sensitivity to VNA S11 amplitude error of 0.01 dB is 22 mK in the high band and 120 mK in the low band respectively and is not strongly influenced by the use of an attenuator.

		Low band	High band
Frequency		50-100 MHz	100-200 MHz
Center		75 MHz	150 MHz
Foreground		1700 K	300 K
0.01 dB		120 mK	22 mK
0.06°	0 dB	70 mK	12 mK
0.06°	6 dB	17 mK	-

Effect of S11 errors

While the effect of LNA phase errors can be reduced by adding attenuation in the low band, attenuation increases the sensitivity to S11 magnitude errors in the high band. Adding a low loss cable delay between the antenna and the LNA could be used in conjunction with averaging over one or more periods of the phase rotation. However the cable has to be extremely uniform of constant impedance to avoid introducing additional sources of error. To avoid significant error compared with those in the table above the attenuator impedance has to be within about 0.05 ohms of 50 ohms. Stated another way the attenuator when terminated with 50 ohms should have S11 below -68 dB. See memos 112, 115 and 117 for a discussion of the requirement for an accurate impedance.