EDGES MEMO #272

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To: EDGES Group

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Subject: Estimates of signature confidence for lowband data using delta Chi-squared boundaries

Errors in non-linear weighted least squares can be estimated from a multidimensional grid search. Confidence limits can be estimated, assuming Gaussian noise, from the contours of Chi-squared

$$\chi^2 = \sum_i \left[ w_i \left( y_i - \sum_k a_k X_k(f_i) \right) \Big/ \sigma_i \right]^2$$

Where $y_i$ = i[th] data point

$w_i$ = weight

$a_k X_k(f_i)$ = k[th] model terms as a function of the i[th] frequency $f_i$

$\sigma_i$ = standard deviation of noise

The minimum value of $\chi^2$ for each of the parameters of interest, which are signature center frequency, amplitude width, and flatness, are obtained using a grid search. For these 4 parameters for each grid point the foreground parameters which minimize $\chi^2$ are found by the standard weighted least squares using the inversion of the design matrix. In the case of the signature amplitude a grid search over 3 parameters (frequency, width and flatness is needed) for each amplitude. Whereas only a 2D grid parameter search is needed for each point of signature frequency, width and flatness since the amplitude can be included along with the foreground terms in a linear weighted least squares solution. Thus a complete four dimensional (4D) search of $\chi^2$ requires three 2D grids plus one 3D grid. For the 4 parameters of interest there are 6 correlations of interest which are each plotted as 2D contours of the difference

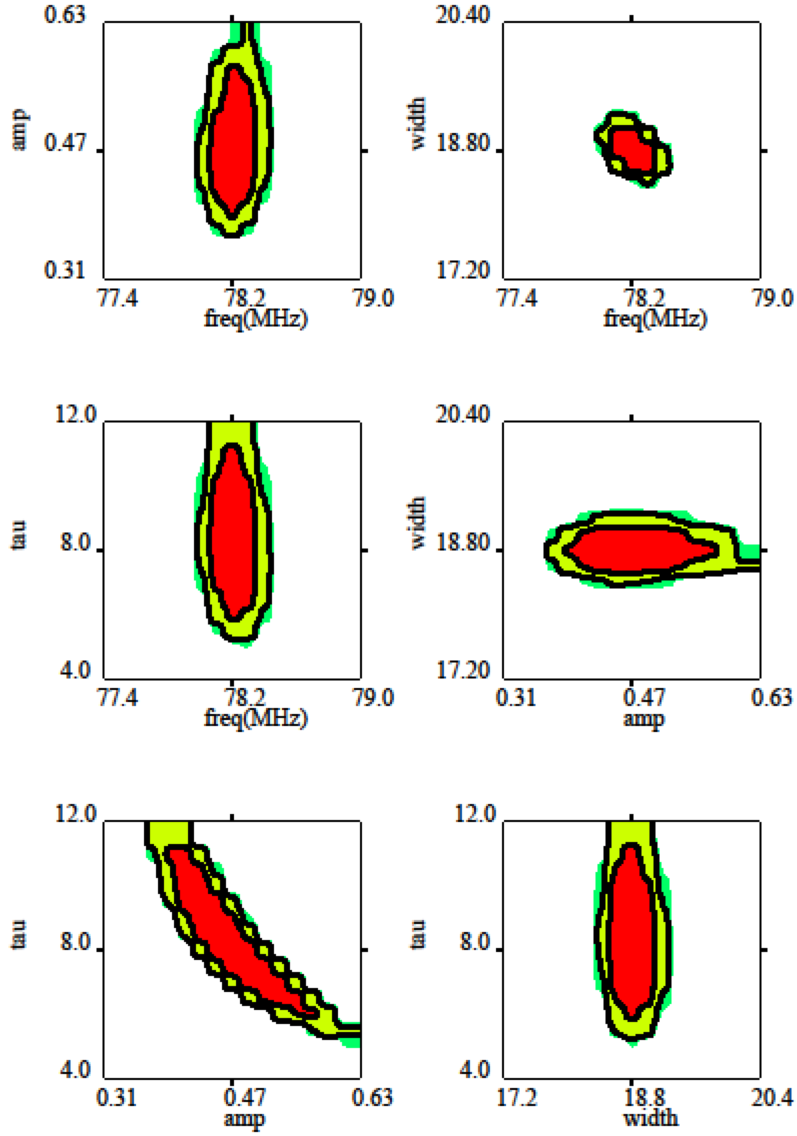
$$\Delta\chi^2 = \chi^2 - \chi_m^2$$

Where $\chi_m^2$ is the minimum value of $\chi^2$ obtained over all parameters. The contours $\Delta\chi^2$ for values of 4 and 9, which represent 95 and 99% confidence are outlined in black so that the regions outside which are white have only 1% probability. Figures 1 and 2 show the results for lowband1 on the extended ground plane and lowband2 EW without balun shield. Each plot used 16 points in each of the 4 dimensions and takes less than 1 minute to run with 100 data points. A uniform weight was assumed and $\sigma^2$ was calculated so that $\chi_m^2 = 1$.

A test of the implementation of this method was made by simulation of a signature of 0.5 k at 78 MHz with width of 19 MHz and $\tau = 7$. 100 mK of Gaussian noise is added to each 400 kHz wide frequency channel. The confidence is estimated for 100 tests using an independent noise stream for each test. The results for range of solutions were

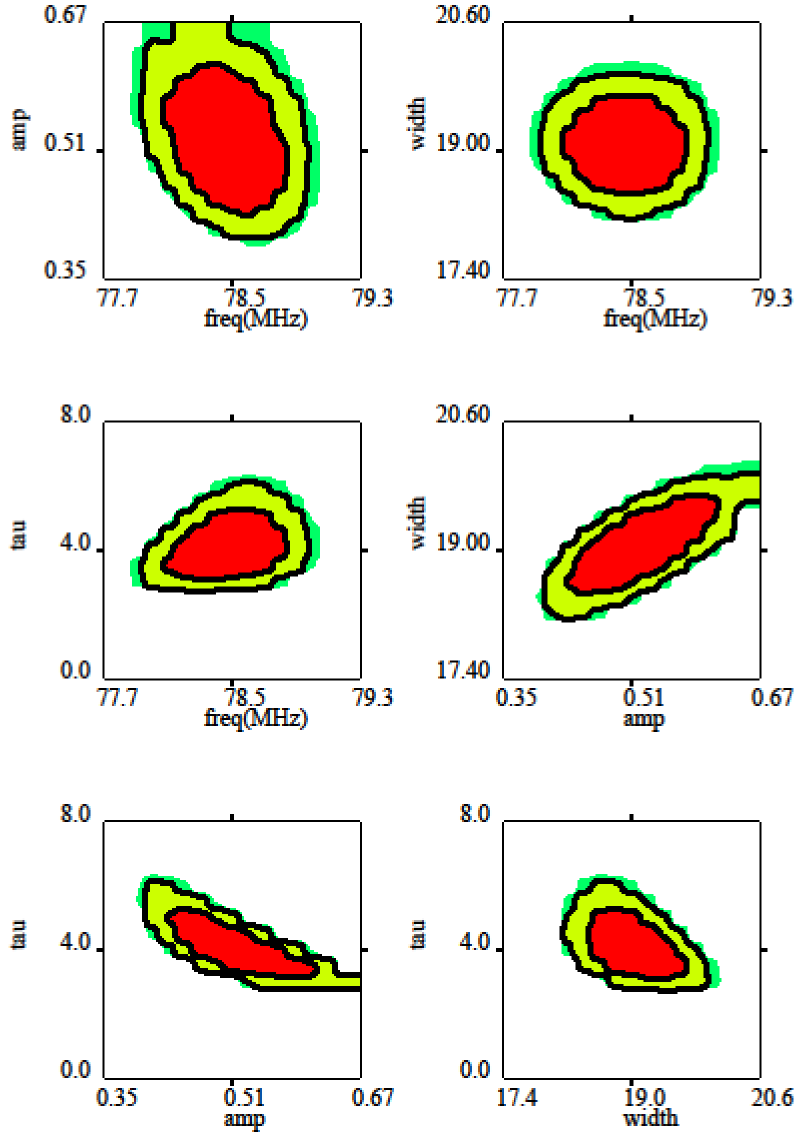
| | |
|---|---|
| Frequency | 77.6 – 78.5 |
| Amplitude | 0.38 – 0.65 |
| Width | 18.0 – 19.8 |
| $\tau$ | 4.5 – 12.0 |

Which with exception of the low limit of 0.38 in amplitude compares quite well with the confidence regions shown in Figure 3 which was obtained for the first instance of the random noise streams.




**fcen 78.20 amp 0.47 wid 18.80 tau 8.00 nterms 5 phy 0 fstart 65.00 fstop 99.00 fsmin 77.00 fsmax 79.00 snr 25.37**

Figure 1. Confidence bounds for lowband1 on extended ground plane.




fcen 78.50 amp 0.51 wid 19.00 tau 4.00 nterms 5 phy 0 fstart 55.00 fstop 99.00 fsmin 60.00 fsmax 79.00 snr 22.60

Figure 2. Confidence bounds for lowband2 EW without balun shield. 2017_180 to 2018_014.

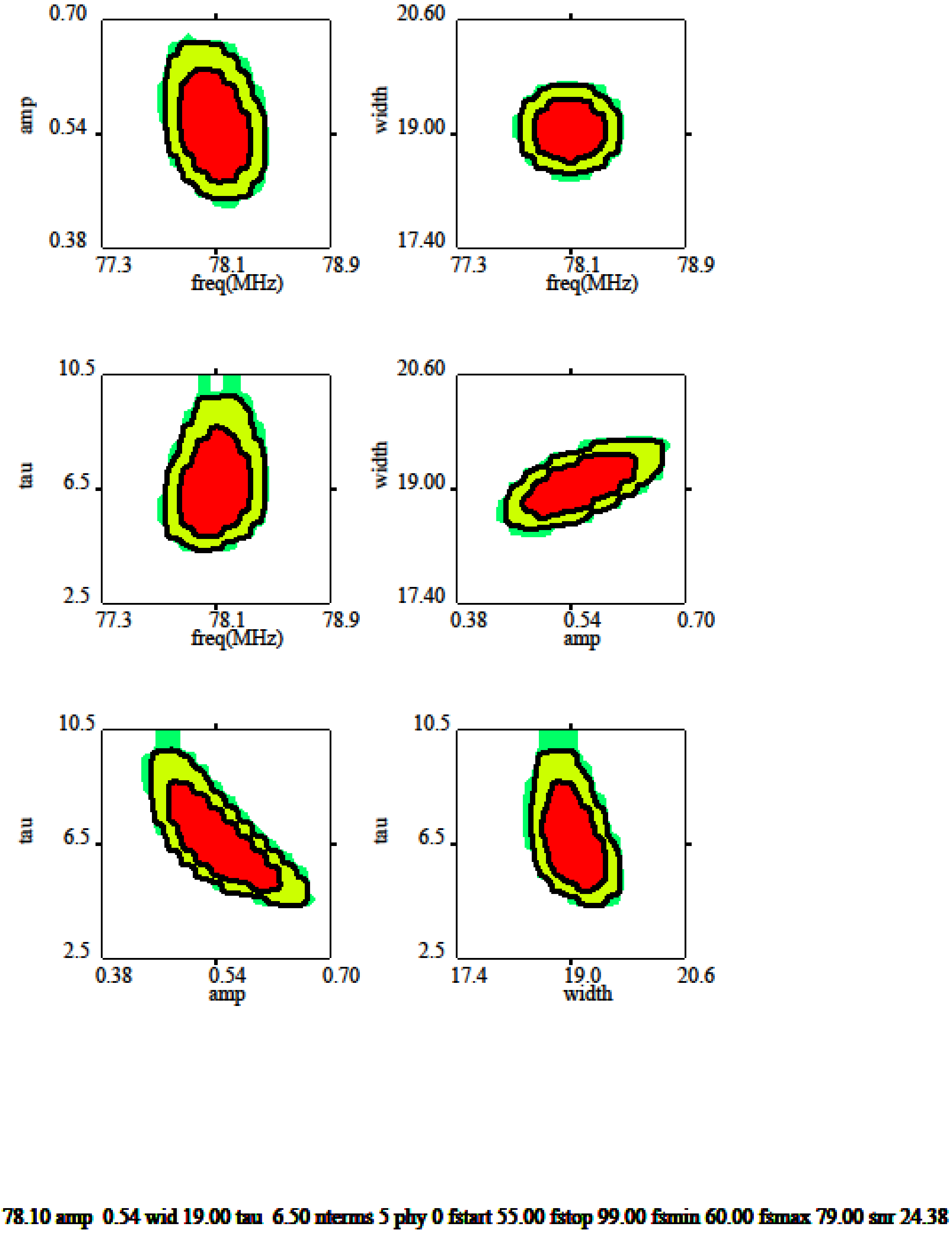



Figure 3. Simulated data with lowband2 S11 and calibration with 0.5 K signature at 78 MHz, width of 19 MHz and $\tau = 7$ added at each frequency.