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To: EDGES Group

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Subject: The effects of refraction by the troposphere and ionosphere

The effects of refraction were studied in memos 101,118, 143,146, 229 and 338. In these simulations the bending action of refraction was considered but the lensing action was ignored. The lensing can be modeled by accounting for the solid angle as in ASU memo 187 using a space based coordinate system. It is found that this method requires a beam pattern with very fine increments of azimuth and elevation. A simpler approach is to move the sky into the fixed azimuth and elevation coordinate frame of the FEKO simulation of the beam. In this case the rays from each pixel of the sky map are moved into the fixed antenna based frame using the refraction and the temperature of map pixel is corrected for the solid angle change lens action by multiplying by the derivative of the bending vs elevation.

For the troposphere the refraction and it's derivative is given by

$$\delta = -1.0/(a + b\Theta + c\Theta^2) \quad (1)$$
$$\frac{d\delta}{d\Theta} = 1 - (b + 2c\Theta)/(a + b\Theta + c\Theta^2)^2 (2)$$

where a=16709.51 b=-19066.21 c=5396.33

δ= refraction angle and Θ= zenith angle in radians

It is found that when this coordinate system is used it is only necessary to interpolate between adjacent beam points in elevation with 1 degree spacing to obtain sufficient accuracy to avoid frequency structure due to computational errors in interpolation. It turns out that the effects of the change in sky map temperature are very small. The much larger effect is that the refracted rays arrive at a higher elevation for which the antenna gain is higher for the horizontal dipole antenna.

The effects of the tropospheric refraction are extremely small and while the effects of the ionospheric refraction increase with the inverse of the frequency squared the frequency structure is very smooth. Table 1 shows the results of the simulations where rms1 and rms2 are the residuals for a 5 term fit and 1 term fit respectively for a frequency range of 52 to 95 MHz. One minute of arc at 45 degrees elevation at 100 MHz with inverse frequency squared dependence is used for the ionosphere. For the ionosphere the refraction and frequency dependency are from the relations in memo 118.

These results show that the refraction effects are very small compared with effects of using a different sky map for the extraction of the absorption. However, in a separate test using EDGES data to derive the temperature scale and offset corrections for the sky maps the effects of refraction are at the level of about 0.0001 in scale and 2K in offset and 0.002 in scale and 40 K in offset at 45 MHz for the troposphere and ionosphere respectively.

case	ground plane	antenna	rms1 min mK	rms1 max mK	rms2 min K	rms2 max K
Tropo refraction	PEC	lowband	1	4	0.09	0.22
Tropo refraction	perf 30x30m	lowband	1	2	0.09	0.22
Ion refraction	PEC	lowband	0	1	2.84	4.86
Ion refraction	perf 30x30m	lowband	1	7	2.88	4.85
Guz vs Has map	PEC	lowband	1 at gha=12	108 at gha=0	1.8	27.5
Guz vs Has map	perf 30x30m	lowband	12 at gha=11	324 at gha=1	1.4	28.8
Guz vs Has map	perf 48x48m	edges-3	2 at gha=8	117 at gha=23	1.2	31.4

Table 1. rms residuals to 5 and 1 term fits to the effects of refraction compared with a change in map

The results in Table 1 also show that the scattering effects of the edges of the 30x30m ground plane result in a significant increase in the rms with 5 terms removed compared with the 48x48m ground plane and the infinite PEC ground plane which appear when the data is simulated with the Guzman 45 MHz map and corrected with the Haslam 408 MHz. It is noted that the effects of the refraction have an extremely small effect when 5 terms are removed because the refraction has a very smooth frequency dependence. However when the effects of scattering or the use of an antenna with complex angular structure, are modeled the angular beam structure convolved with the angular structure produced by the ionosphere can result in fine frequency dependent structure. For example Table 2 shows the results of simulations of the effect of the refraction using FEKO beam models for other types of antennas.

case	ground	antenna	rms1 min mK	rms1 max mK	rms2 min K	rms2 max K
Ion refraction	soil	conical log spiral	10	32	3.5	6.9
Ion refraction	mesh	inverted-V dipole	2	19	3.8	6.7
Ion refraction	lake	vertical monopole	0	2	8.6	18.0

Table 2. Residuals to 5 and 1 term fits to the effects of refraction for other antennas.

The beam model for the conical log spiral antenna is for an antenna of 0.7m in height in an attempt to find an antenna as good as or better than EDGES. The results of the inclusion of the absorption and emission of the ionosphere have a very small effect of the 5 term fits owing to the extreme smoothness of absorption and emission. The absorption and emission do make large changes to the 1 term fits.

Figures 1 and 2 show plots of the simulated spectra for one hour blocks of GHA for the case of the effect of ionospheric refraction on the lowband antenna on the 30x30m ground plane. Note the very fine scale in plots of Figure 1 which are needed to show the small effects when 5 terms are removed. These small effects which effect the extraction of the global 21-cm signal are quite significant for the conical log spiral antenna and less significant for the inverted-V dipole. The vertical monopole has very low sensitivity to the ionosphere but needs a very large ground plane to avoid the effects of scatter from surrounding objects as discussed in memos 340, 341, 344, 345 and 348.

Differences in EDGES-2 high band spectra taken at the same GHA on different days shows the smooth nature of the effect of the ionosphere in memo 143. These spectra differences are used to estimate the electron temperature in the ionosphere a function of local time and are reported in
Rogers, A. E. E., Bowman, J. D., Vierinen, J., Monsalve, R. and Mozdzen, T. (2015). Radiometric measurements of electron temperature and opacity of ionospheric perturbations. Radio Sci., 50, 130–137. doi: 10.1002/2014RS005599

Differences in EDGES-2 low band band spectra taken at the same GHA on different days are
analyzed for data for 2017 days 120 to 129 in Figure 3 and 2020 days 120 to 129 in Figure 4. In both cases show the difference of the spectra for the days indicated of the right hand side along with the best fit value for the opacity difference first minus the second day and electron temperature which is assumed to be the same for each day followed by the rms residual for the 2-term fit. The difference spectra have been smoothed to 1.56 MHz resolution to reduce the noise which dominates the residuals owing to the relatively short integrations of 30 minutes.

In summary these simulations show that refraction is not a significant source of error in the extraction of global 21-cm signal compared with scattering effects and instrumental errors and the relatively smooth effects of changes in the ionosphere seen in the EDGES high and lowband data provide some confidence that the refraction, absorption and emission of the ionosphere is not a limiting factor in extracting the global 21-cm absorption.

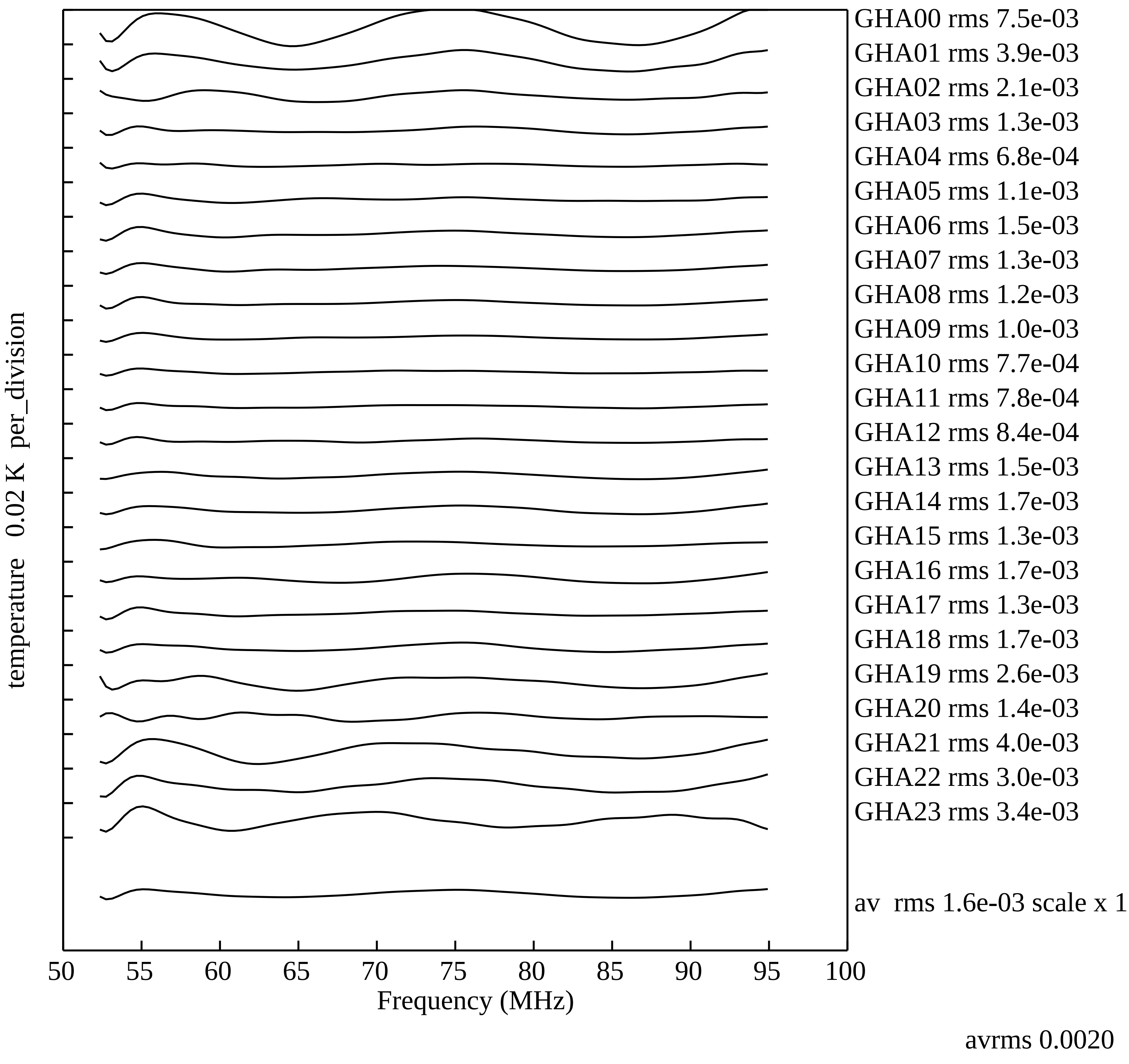

Figure 1. Plot of residuals of ionospheric refraction for 5-terms removed for lowband antenna on 30x30m ground plane 4th entry of table 1,

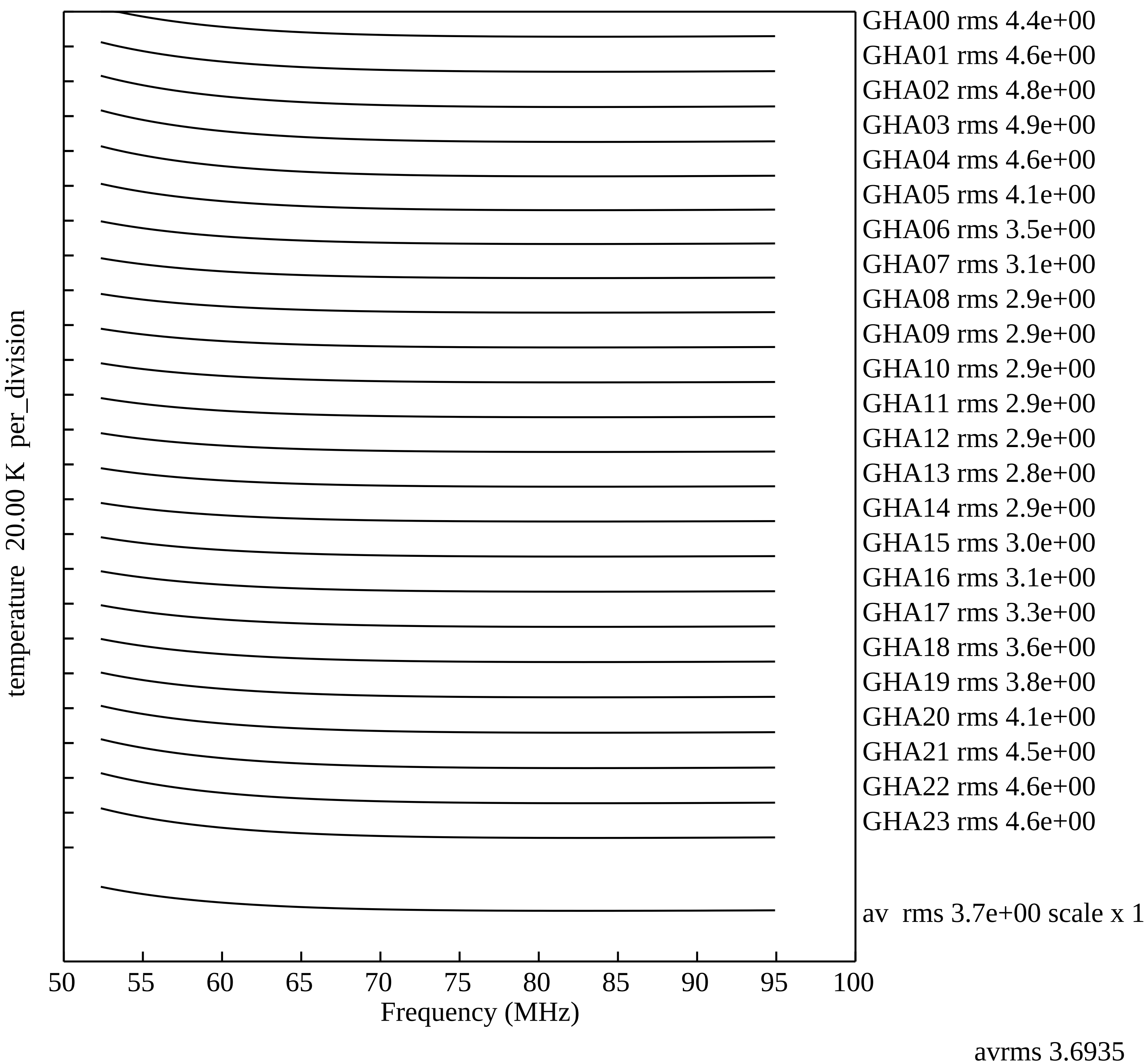

Figure 2. Plot of residuals of ionospheric refraction for 1-term removed for lowband antenna on 30x30m ground plane 4th entry of table 1.

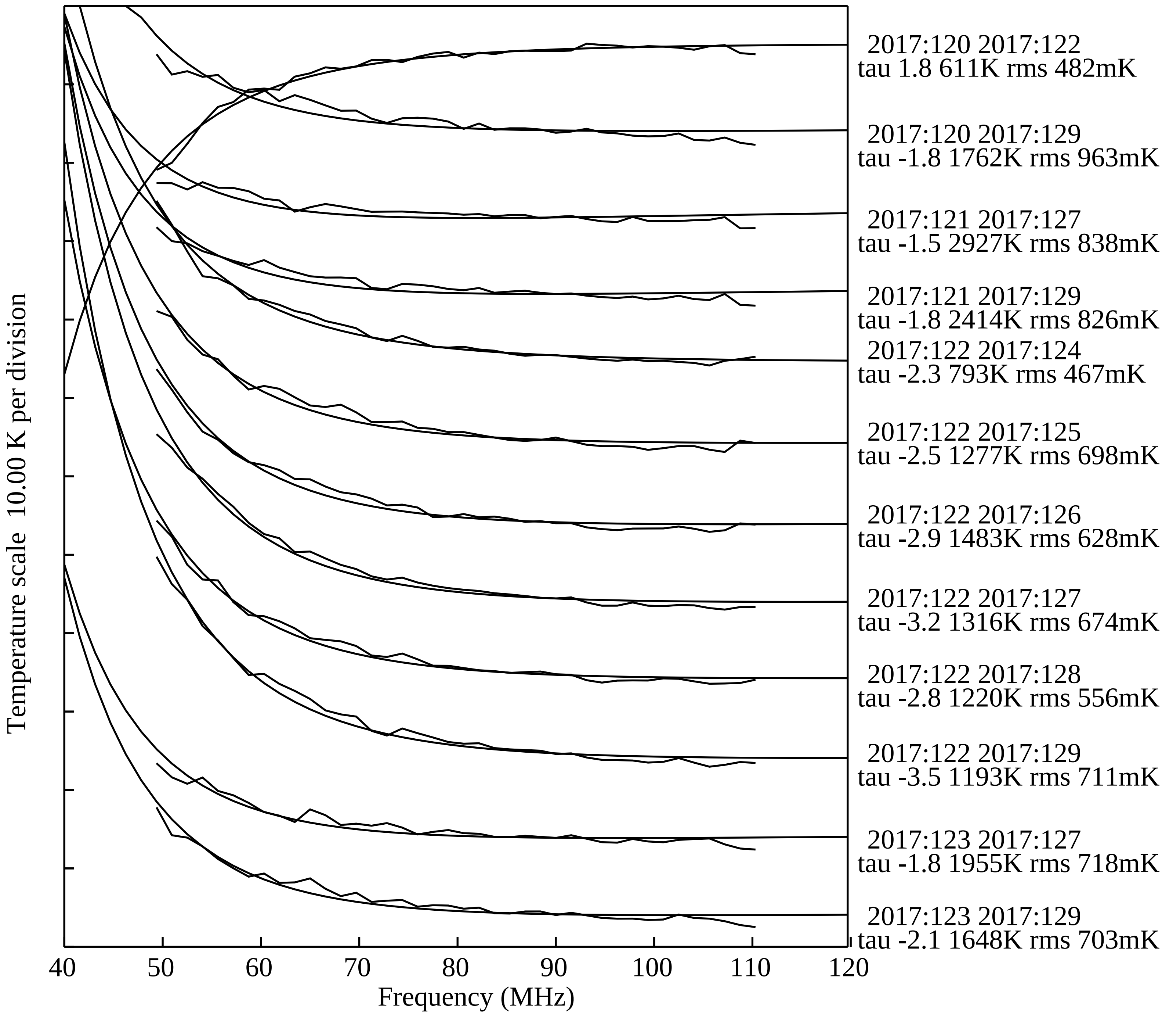

Figure 3. Examples of the difference spectrum of the largest ionospheric perturbations at GHA of 10 h for lowband data in 2017 days 120 to 129. The dates of the two spectra whose difference was taken are given on the right of the plot along with the change in opacity at 75 MHz in units of parts per thousand.

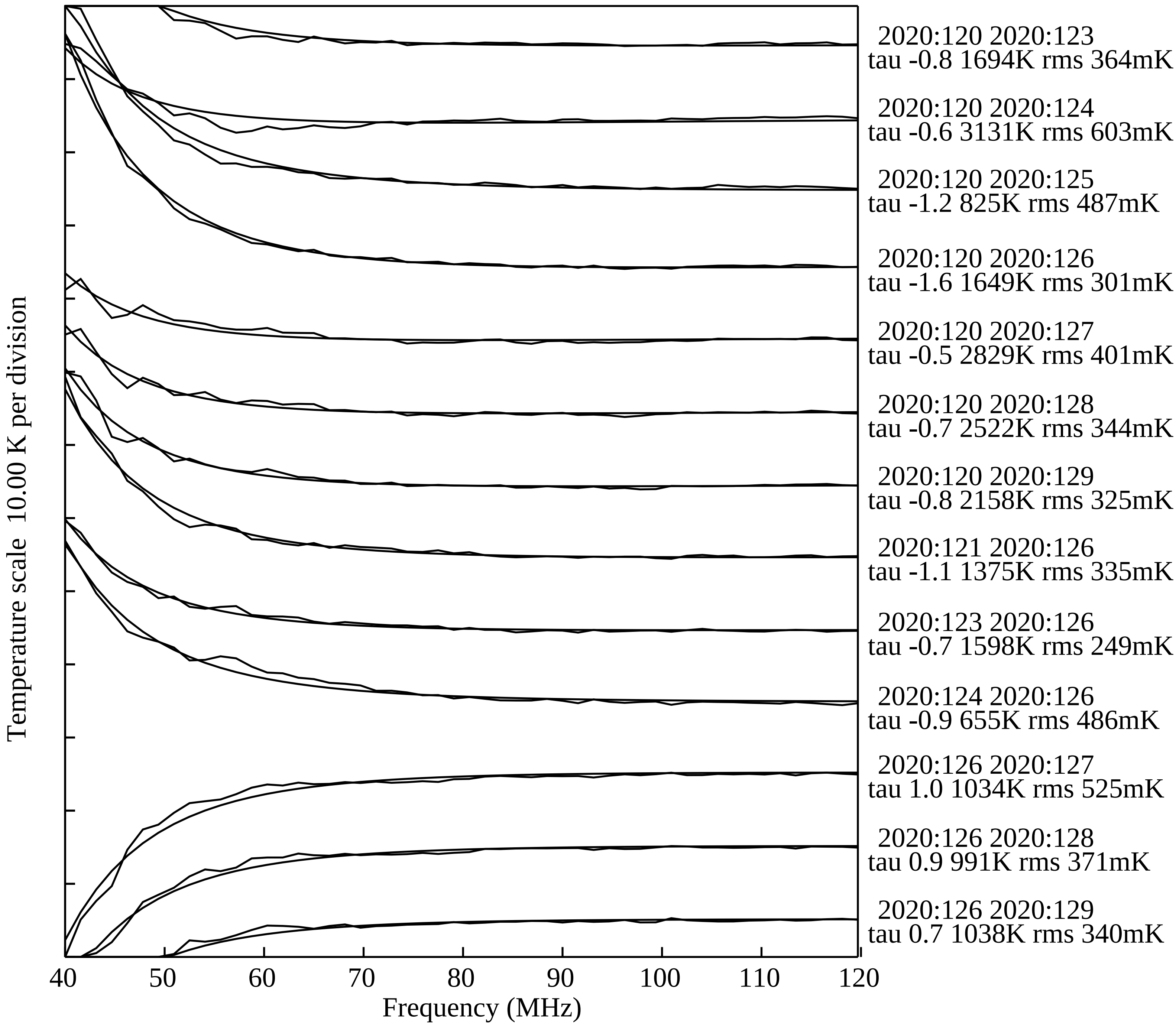

Figure 4. Examples of the difference spectrum of the largest ionospheric perturbations at GHA of 10 h for lowband data in 2020 days 120 to 129. The dates of the two spectra whose difference was taken are given on the right of the plot along with the change in opacity at 75 MHz in units of parts per thousand.