SRT MEMO #018

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To: SRT Group

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Subject: Interferometer geometry calculations

For the "VLBI" mode we start with the latitude, longitude and height of each end of the "baseline" and convert to geocentric right handed *x, y, z* coordinates. This coordinate conversion is done by function

$$x = (n + hgt)\cos(lat)\cos(lon)$$
$$y = (n + hgt)\cos(lat)\sin(lon)$$
$$z = (n + (1-e) + hgt)\sin(lat)$$

where $n = a \Big/ \left(1 - e\sin^2(lat)\right)^{1/2}$

a = 6378137 m WGS84
e = 2f-f ^{2}
f = 1/298.257223563 WGS84

The vector baseline is defined as the vector from site1 (the "reference" site) to site2 (the remote site)

$$b_x = x_2 - x_1$$
$$b_y = y_2 - y_1$$
$$b_z = z_2 - z_1$$

The delay τ of a signal's arrival at the remote site is $\tau = -\vec{b} \bullet \hat{s} \big/ c = -\left(b_x s_x + b_y s_y + b_z s_z\right) \big/ c$

Where c = velocity of propagation
$\hat{s}$ = unit vector in the direction of the source
s_x = cos (dec) cos (gha)
s_y = -cos (dec) sin (gha)
s_z = *sin(dec)*

where *gha* = *gst* – *ra* = Greenwich hour angle
gst = Greenwich sidereal time

ra = apparent right ascension
dec = apparent declination

or from the derivatives of the phase with respect to r*a* and dec

$$\phi = (2\pi/\lambda)\left(\cos(dec)\cos(gha)b_x - \cos(dec)\sin(gha)b_y + \sin(dec)b_z\right)$$

In units of fringes per arc second

$$u = \left(b_x \sin(gha) + b_y \cos(gha)\right)(\pi/648{,}000\lambda)$$
$$v = \left(b_z \cos(dec) - b_x \cos(gha)\sin(dec) + b_y \sin(gha)\sin(dec)(\pi/648{,}000\lambda)\right)$$

The interferometer phase (normally defined as being positive (NRAO's convention) when the signal arrives earlier at the 2nd site is

$$\phi = +2\pi\vec{b}\bullet\hat{s}/\lambda \quad (radians)$$

or $\phi = -2\pi\tau f \quad (radians)$
where λ = wavelength (m)
f = frequency (Hz)

The components of the baseline projected in the direction of the source in the directions of increasing RA and increasing declination are known as *u* and *v* and are often expressed in units of fringes per arc second. These can be derived from the baseline projections

$$u = b_x \sin(gha) + b_y \cos(gha)$$
$$v = b_z \cos(dec) - b_x \cos(gha)\sin(dec) + b_y \sin(gha)\sin(dec)$$