EDGES MEMO #368

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To: EDGES Group

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Subject: Dependence of sensitivity to VNA delay error on antenna and LNA s11

Tests of the effects of VNA delay error on EDGES-3 have been simulated using the measure calibration file along with the simulated antenna spectrum and antenna s11 from FEKO. These results are shown in the table in Figure 1. The large sensitivity to a s11 delay error of 100 ps for the antenna and a similar sensitivity and effect of 100 ps in the s11 of the LNA has prompted a closer examination of the origin and better understanding of the effects based on the equation for the noise power from a LNA connected to the antenna.

$$P = [T_a(1 - |\Gamma_a|^2 + T_u|\Gamma_a|^2)]|F|^2 + |\Gamma_a||F|(T_c\cos\phi + T_s\sin\phi) + T_0$$

where $F = (1 - |\Gamma_l|^2)^{1/2}/(1 - \Gamma_a\Gamma_l)$
and $\phi = phaseof(\Gamma_a F)$

P = power from LNA
Ta = antenna noise temperature
Tu = uncorrelated noise
Tc and Ts = cosine and sine components of correlated noise
To = noise which is taken out in the 3-position switching
Γa = antenna s11
Γ_l = LNA s11

Previously memos 262 and 321 have shown the large sensitivity to LNA s11 delay error. The term in the product of the antenna and LNA s11 in F has been identified as the term in the EDGES analysis which is very sensitive to VNA delay error.

I find that magnitude of the effect of the error is approximately given by

$$2 * |\Gamma_a| * |\Gamma_l| * 2\pi * frequency * (antennadelayerror + lnadelayerror) * Tsky$$

which is approximately 500 mK for 100 ps error in both antenna and lna S11 for $|\Gamma a|$ = -20 dB, $|\Gamma_l|$ = -40 dB Tsky = 3000 at 75 MHz. The ripple period being approximately the inverse of the typical 20 ns in the antenna plus balun delay or about 50 MHz. This result is in approximate agreement with simulations.

Low reflection coefficients for the antenna and LNA are needed to reduce the effects of VNA measurement delay error. It is also noted that errors in the delay of the antenna and LNA s11 add. Errors in VNA amplitudes tend to have a smoother effect compared with the phase errors in the product of the antenna and LNA s11. The result is that s11 phase error is the main source of instrumental error in the measurement of the 21-cm absorption. The errors in the estimation of the noise waves and errors in s11 magnitude have a much smaller effect. Going to an electrically small antenna does reduce the delay in the antenna s11 which makes a smoother error but the s11 is increased. It is also noted the elimination of this error source, which arises the sky noise reflecting back and forth (see memo 76) between the LNA and antenna, requires all 21-cm systems to very accurately measure the phase of the antenna and LNA s11.

EDGES-3 sensitivity to possible calibration errors

Error source	uncertainty	modeled	recovered parameters				rms1	rms2
			center	snr	amp	width		
LNA S11 magnitude	0.01 dB	1.0 dB	77.4	22	0.53	20.3	53	22
LNA S11 delay	20 ps	100ps	78.5	19	0.37	19.0	38	19
Antenna S11	0.002 dB	0.02 dB	77.7	42	0.60	19.0	65	15
Antenna S11 delay	20 ps	100ps	78.5	29	0.40	19.0	41	14
No loss correction			78.1	109	0.48	19.0	47	4
No beam correction			78.5	31	0.46	18.0	53	16

Simulations using 5 physical terms removed 53 – 95 MHz GHA = 12hrs at the MRO site on 48x48m ground plane. rms1 and rms2 are residuals in mK with Nature feature added to foreground and after solving for feature with fixed tau = 7.

a) Power from a LNA connected to a mismatched load

$$P = \left[T_a \left(1 - |\Gamma_a|^2\right) + T_u |\Gamma_a|^2 \right] |F|^2 + |\Gamma_a| |F| \left(T_c \cos\phi + T_s \sin\phi \right) + T_\ell$$

where $F = \left(1 - |\Gamma_\ell|^2\right)^{1/2} \Big/ \left(1 - \Gamma_a \Gamma_\ell\right)$

and $\phi = \text{phase of } \left(\Gamma_a F\right)$

term which dominates with delay error