VGOS MEMO #055

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To: IVS VGOS schedulers and analysts
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Subject: VGOS band SNRs from channel fringe visibilities

0. Summary

A formula is presented for calculating SNRs in individual VGOS frequency bands using as input only quantities available in a vgosDB: channel fringe ampitudes and phases and the total SNR and fringe amplitude for the four bands combined. Band SNRs calculated with the formula for VO0357 data agree with those from running *fourfit* on a single band to <1% in most cases. Agreement is worse at low SNR.

1. Formula

Let SNR_{band} = SNR for a single band with N frequency channels (usually, N = 8)
SNR_{tot} = SNR for the combined four bands with a total of M channels (usually, M = 32)
amp_{tot} = coherent average fringe amplitude for the combined four bands from *fourfit*
$\mathbf{V}_j$ = complex fringe visibility of channel j

Fringe amplitude and phase are related to $\mathbf{V}_j$ by $|\mathbf{V}_j|$ = amplitude and $\arg(\mathbf{V}_j)$ = phase residual to *fourfit* model.

The formula is $SNR_{band} = [\,(SNR_{tot} / amp_{tot}) \times (N/M)^{1/2}\,] \times [\,|\sum_{n=1}^{N} \mathbf{V}_n| \,/\, N\,]$, where the terms inside the first set of square brackets convert band fringe amplitude to SNR, and the expression inside the second set of brackets is the coherent average amplitude of the channel visibilities in the band of interest. Generally, SNR and amplitude are related by SNR = constant × amplitude × $\sqrt{\#\text{samples}}$. Under the assumption that all channels have the same number of samples,[1] the total number of samples in a band is N/M times smaller than in all four bands, hence the band amplitude-to-SNR conversion factor is $(N/M)^{1/2}$ times smaller than the four-band factor. Combining the two factors involving M and N yields

$$SNR_{band} = (SNR_{tot} / amp_{tot})\ \left|\sum_{n=1}^{N} \mathbf{V}_n\right| \,/\, (MN)^{1/2} \tag{1}$$

An alternative, more symmetrical formula based on equation (1), with amp_{tot} replaced by the magnitude of the coherent average channel visibilities over all M channels, was suggested by John Gipson:

$$SNR_{band} = SNR_{tot}\ \left|\frac{\sqrt{M}}{\sum_{m=1}^{M} \mathbf{V}_m}\right|\ \left|\frac{\sum_{n=1}^{N} \mathbf{V}_n}{\sqrt{N}}\right| \tag{2}$$

2. Test of formula

The accuracy of equation (1) was tested by fringe-fitting, one band at a time, all 2285 observations from the first six hours of VGOS session VO0357 for six stations and then comparing the band SNR values

[1] This assumption is justified when recording VLBI data on hard disks or solid state drives. In the olden days of tape recording, however, track-dependent playback problems could lead to significant differences, from channel to channel, in the amount of recorded data recovered at the correlator.

with those calculated with equation (1) from the original, 4-band, production *fourfit* output. The six stations were GGAO12M, KOKEE12M, MACGO12M, ONSA13SW, WESTFORD, and WETTZ13S. The production *fourfit* control file was used for all fringe-fitting, with the only difference in *fourfit* runs being the selection of which frequency channels to include.

Because SNRs and amplitudes are biased slightly high by noise, especially at low SNR, the single-band SNRs from *fourfit* were corrected, on average, for this effect by dividing by $1 + 1/(2\ SNR^2)$ (Thompson, Moran and Swenson, *Interferometry and Synthesis in Radio Interferometry*, 3rd edition, equation 9.65).

Figure 1 shows the distribution, by frequency band, of the band SNR ratios in the sense of "predicted" (i.e., calculated from 4-band *fourfit* output using the formula) divided by "observed" (i.e., from single-band *fourfit* output), or "formula" / "truth".

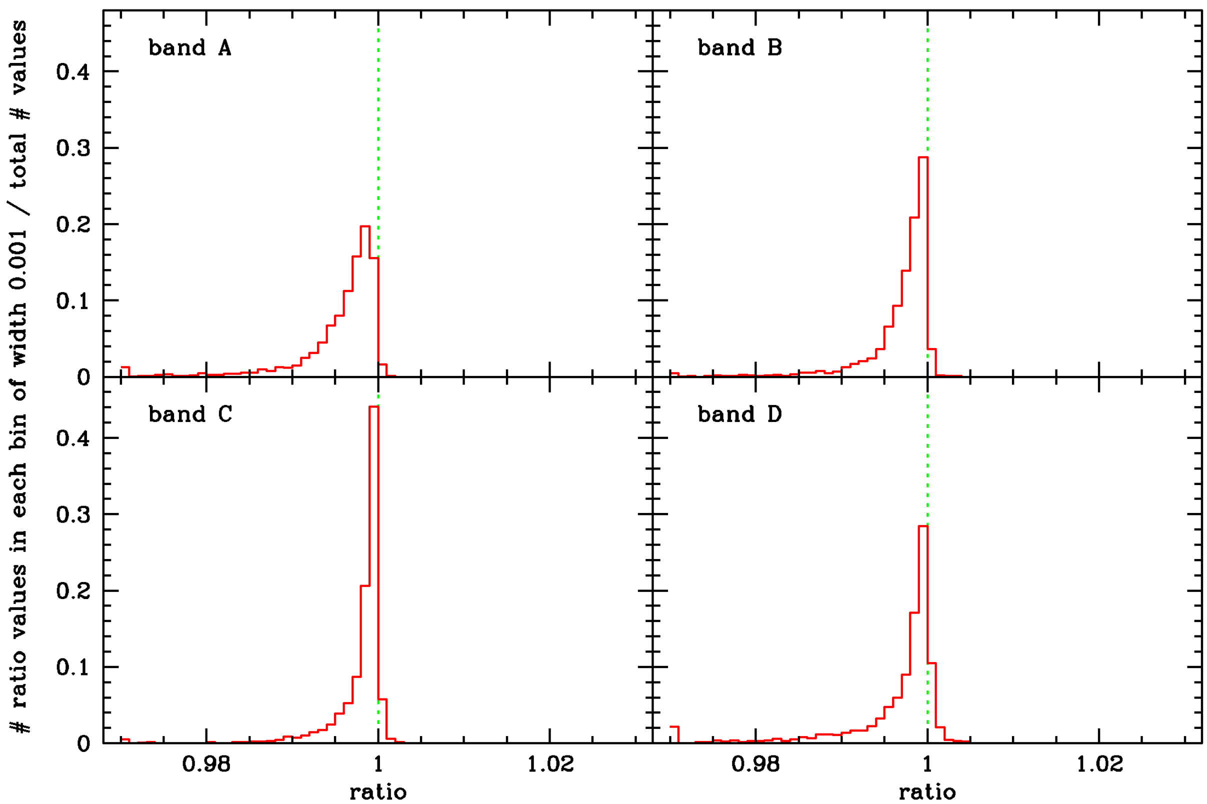

Figure 1. Distribution by band of ratio between band SNR calculated from equation (1) and band SNR from *fourfit*, for observations among 6 stations during first 6 hours of VO0357. Only observations with *fourfit* QC ≥ 5 are included. Bin width = 0.001 (0.1%). The last bin on the left in each panel includes all ratios below 0.97.

Two important features of Figure 1 are (1) the fact that the great majority of ratios (93% of all ratios in the four bands) are within 1% of the desired value of unity, and (2) the asymmetry, or skewness, in the distributions, with longer tails below unity than above. The asymmetry is in the sense that the formula SNRs tend to be too low, on the assumption that the SNRs from fringe-fitting one band are "true."

Neither the asymmetry nor the fact that the ratios are not exactly unity should be a surprise, given the design of *fourfit* and differences in how channel and coherent average fringe amplitudes are estimated. For example, coherent average amplitudes are subject to small corrections that are not applied to channel amplitudes. Also, because global *fourfit* parameters (multiband delay, singleband delay, etc.) inevitably

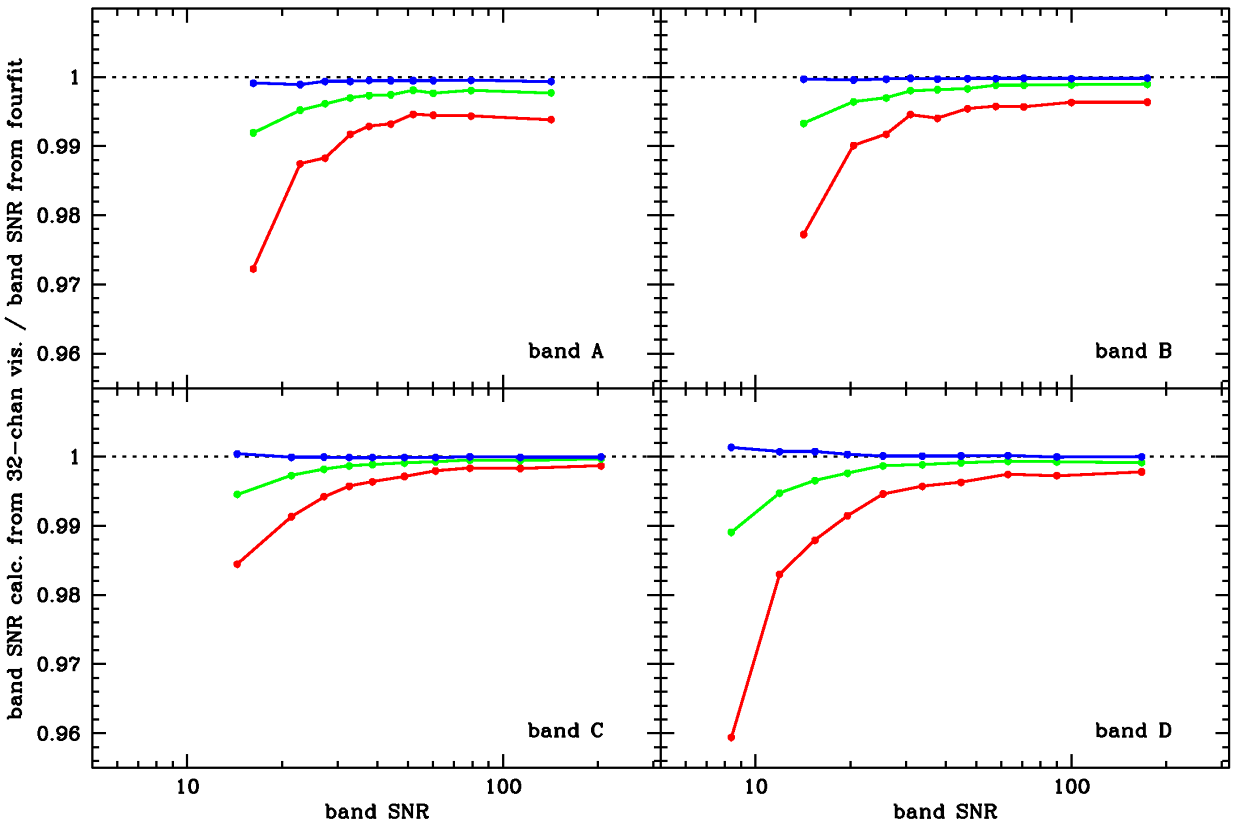

Figure 2. 10th (red), 50th (green) and 90th (blue) percentile points of ratios in Figure 1 vs. band SNR. Data were binned by SNR into 10 bins, each containing 220-229 observations, and the 10th, 50th (median) and 90th percentile points of the ratios in each bin were computed. Each percentile point is plotted at the median SNR for the bin.

differ when fringe-fitting one and four bands, the channel amplitudes come out differently. No attempt has been made to understand quantitatively the nature of the distributions in Figure 1.

In the future, scheduled durations of VGOS observations will almost certainly be shorter than the current nominal value of 30 seconds, and SNRs will accordingly be lower. In order to test how well the formula performs at lower SNRs, the 10th, 50th (median), and 90th percentile points of the ratio distributions are plotted versus SNR in Figure 2. The median ratio (green dots) lies between 0.99 and 1.00 for all SNR > 10, and it dips slightly below 0.99 only at SNR ~ 8 in band D. The 10th percentile points are significantly lower, reaching a minimum of ~0.96 at SNR ~ 8 in band D. In comparison, the 1-σ formal error of SNR is ~1, which means the SNR estimated either by formula or by *fourfit* will scatter by a fractional amount ~1/SNR, or ~10% at SNR ~ 10. This scatter is much larger than the median biases seen in Figure 2, although of course biases cannot be reduced by averaging multiple observations.

Band SNRs from equation (2) differ from equation (1) SNRs by the factor $\mathrm{amp_{tot}} / \frac{1}{M} \left| \sum_{m=1}^{M} \mathbf{V}_m \right|$. The vast majority of the VO0357 values for this factor lie between 0.999 and 1.005, even at lower SNRs. For instance, for $\mathrm{SNR_{tot}} < 30$, 91% of the values lie in that range.

3. Slightly better formula

The performance of either equation can be improved slightly by correcting for a possible slope in channel fringe phase vs. frequency, which can cause loss of phase coherence across the band. (When *fourfit* is run on all four bands, global phase gradients across the M channels are removed by estimating multiband and ionospheric delays and then removing their contributions to the channel phases. But linear phase gradients may still exist over the channels in a single band within the four bands due to, e.g., source structure phases or imperfect additive phasecal corrections.) Linear phase gradients in a band can be removed by calculating the amplitude of the gradient-adjusted coherently summed visibilities $|\sum_{n=1}^{N} \mathbf{V}_n\ e^{-i2\pi\nu_n\tau_{trial}}|$ at several trial delay values (ν_n is the frequency of the n[th] channel), then interpolating over delay to find the delay τ_{max} at which the amplitude reaches its maximum.

Figure 3 shows the band SNR ratio percentiles with $|\sum_{n=1}^{N} \mathbf{V}_n|$ in equation (1) replaced by $|\sum_{n=1}^{N} \mathbf{V}_n\ e^{-i2\pi\nu_n\tau_{max}}|$. Compared with Figure 2, the improvements in the agreement between predicted and observed band SNRs are minor, with the largest improvement occurring at low SNR in band D. For such a small improvement, the additional computations needed to remove the phase gradients may not be warranted.

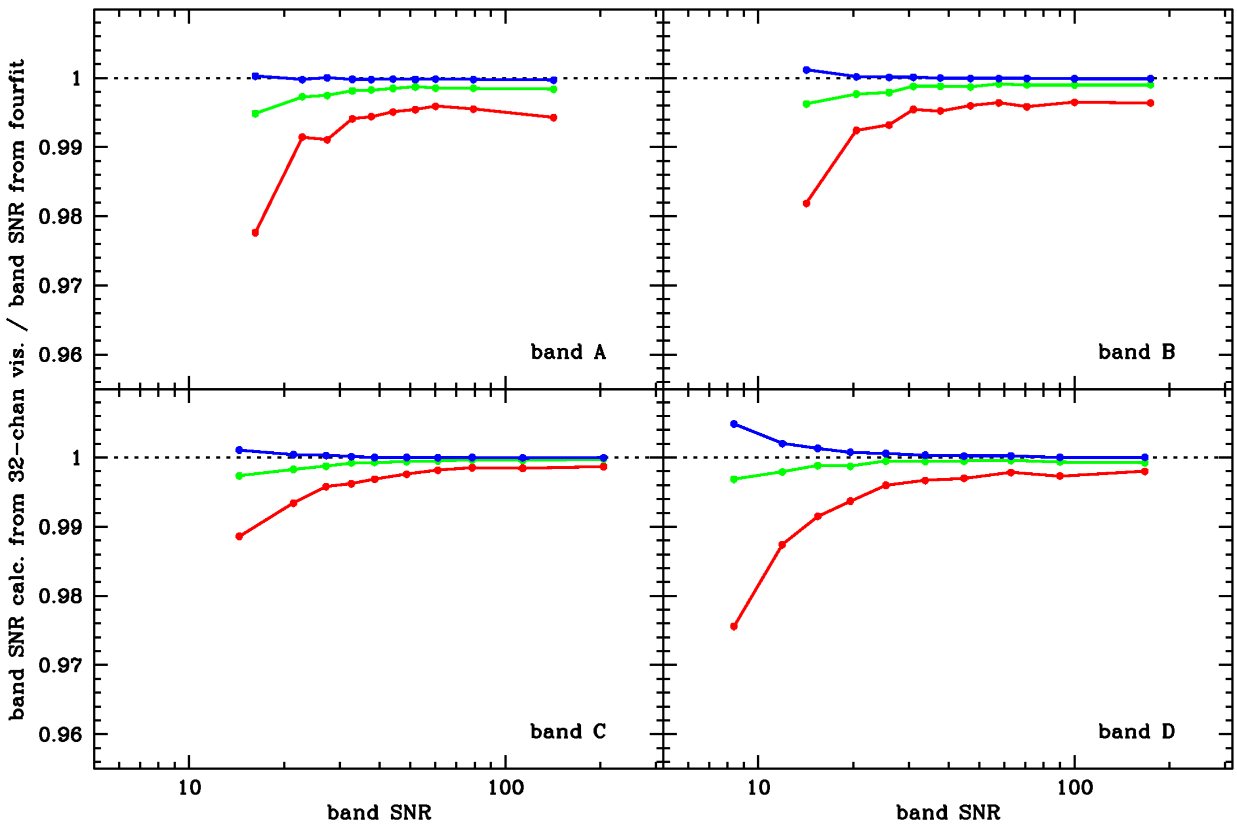

Figure 3. Same as Figure 2 except that phases of visibilities in eq. (1) have been corrected for group delay.